$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)}$$

Likelihood

Prior

Evidence

Posterior

$$P(y) = \sum P(y|x_i)\,P(x_i)$$

You might suppose

$$P(x_t|y_t) = \frac{P(y_t|x_t)\,P(x_t)}{P(y_t)}$$

State measurement

Process noise

$$x_{t+1} = f(x_t, w_t)$$

Sensor Noise measurement

$$y_t = f(x_{\cancel{t}}) + v_t$$

$$P_r(y_t|x_t) \Leftarrow y_t = f(x_t) + v_t$$

The statistics of the measurement are determined by the statistics of the noise.

$\boxed{\text{Expected Value}}$ (mean)

$$\hat{x} = E\{x_t\}$$
$$= \sum_i x_t^i\,\hat{P}_r(x_t^i|y_t)$$

sometimes

Strength in Bayes Processing

+ Simulation

+ data-based approaches

Syllabus

- Buy Candy's book → "Bayesian Signal Processing"

INTRO

- Estimation of signals & parameters of systems
  - things screw with signals!
    - noise
    - disturbances
  - of underlying probability distributions

Some signal

$$X \sim \Pr(X|Y)$$

Probability

Data / Measurements

Signal (Random Variable)

Sampled from or dist according to

The Problem?

We don't always know $\Pr(-)$!

We usually estimate it $\underbrace{\hat{\Pr}(X|Y) \Rightarrow \hat{X} = \arg\max_X \hat{\Pr}(X|Y)}_{\text{Maximum Likelihood Estimation}}$

How does $\hat{\Pr}(X|Y)$ change w/ new data?

$\boxed{\text{Bayes Rule}}$

Suppose we're given/know $P(X)$ ← Prior knowledge

- We get some data, Y

  we want $P(X|Y)$

Suppose we want to evaluate the integral

$$I = \int_a^b h(x)\,dx \quad x \in \mathbb{R}^p$$

→ could use Riemann integral

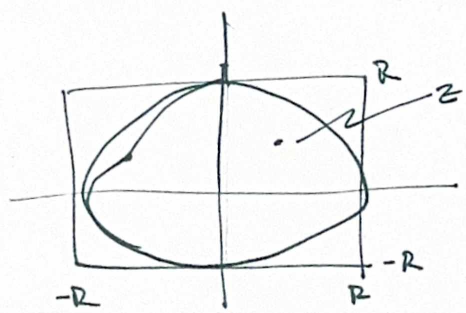
alternate method

$$I = \int_a^b h(x)dx = \int_a^b w(x)\,p(x)\,dx = E\{w(x)\}$$

$$p(x) = \underbrace{\frac{1}{b-a}}_{\text{uniform dist}} \rightarrow w(x) = h(x)(b-a)$$

"Flat-top"

$$I = \frac{1}{N}\sum_{i=1}^{N} w(x_i) \quad x \sim p(x) = U(a,b)$$



$z = (x, y)$
$x \sim U(-r, r)$
$y \sim U(-r, r)$

$$P(\text{w/in circle}) = \frac{\#\text{ in circle}}{\#\text{ samples}}$$

$$P(\text{w/in circ} | sq) = \frac{P(sq|cr)\,P(cr)}{P(sq)}$$

$$= \frac{\pi r^2}{4r^2} = \frac{\pi}{4} = \frac{k}{N}$$