Xenon Transient Information Oct 2024 Rev 2
W. N. Locke

When fission products build up in the core, they will significantly impact reactor operation. As a poison concentration goes up it will add negative reactivity. The core power will go down below steam demand causing the system to cool adding positive reactivity. The power will be restored to steam demand but at a lower temperature. Action will be required to restore temperature. This will either be pulling rods or diluting boron.

This document provides some example transient calculations for a large thermal power reactor. The data here is drawn from information on the Westinghouse SNUPPS reactors near the beginning of life (BOL).

I-135 fission yield	γ_{I}	5.7%
Xe-135 fission yield	γ_{Xe}	0.3%
I-135 decay constant (6.7 hour $t_{1/2}$)	λ_I	2.87e-05 sec^{-1}
Xe-135 decay constant (9.2 hour $t_{1/2}$)	λ_{Xe}	2.09e-05 sec^{-1}
Full Power Burnout Factor $\sigma_a^{Xe}\varphi_{th}^{100\%}$	R^{Max}	7.34e-05 sec^{-1}
Power Constant based on a Full Power Equilibrium Xe Reactivity of -2900 pcm	K	4.56 pcm x sec^{-1}

With the following equations the units of the number densities N_I, and N_{Xe} are pcm's. The equations are presented for review and for discussion. The quantity, p, varies from 0 to 1 as power varies from 0% to 100%.

Xenon and Iodine differential equation

$\mathrm{A} = \begin{bmatrix} -\lambda_{\mathrm{I}} & 0 \\ \lambda_{\mathrm{I}} & -\lambda_{\mathrm{Xe}} - \mathrm{pR^{Max}} \end{bmatrix}$	$\mathrm{B} = \mathrm{pK}\begin{bmatrix} \gamma_{\mathrm{I}} \\ \gamma_{\mathrm{Xe}} \end{bmatrix}$
$\frac{\mathrm{dX}}{\mathrm{dt}} = \mathrm{AX} + \mathrm{B}$	$\mathrm{X} = \begin{bmatrix} \mathrm{N_I} \\ \mathrm{N_{Xe}} \end{bmatrix}$

Equilibrium values are as follows

$X^{Eq} = -A^{-1}B$	$A^{-1} = \begin{bmatrix} -\lambda_{\mathrm{Xe}} - \mathrm{pR^{Max}} & 0 \\ -\lambda_{\mathrm{I}} & -\lambda_{\mathrm{I}} \end{bmatrix} * \frac{1}{\lambda_{\mathrm{I}}(\lambda_{\mathrm{Xe}} + \mathrm{pR^{Max}})}$
$N_I^{Eq} = \frac{\gamma_{\mathrm{I}} K p}{\lambda_{\mathrm{I}}}$	$N_{Xe}^{Eq} = \frac{(\gamma_{\mathrm{I}} + \gamma_{\mathrm{Xe}}) K p}{\lambda_{\mathrm{Xe}} + \mathrm{pR^{Max}}}$

Iterate solution based on a first order difference

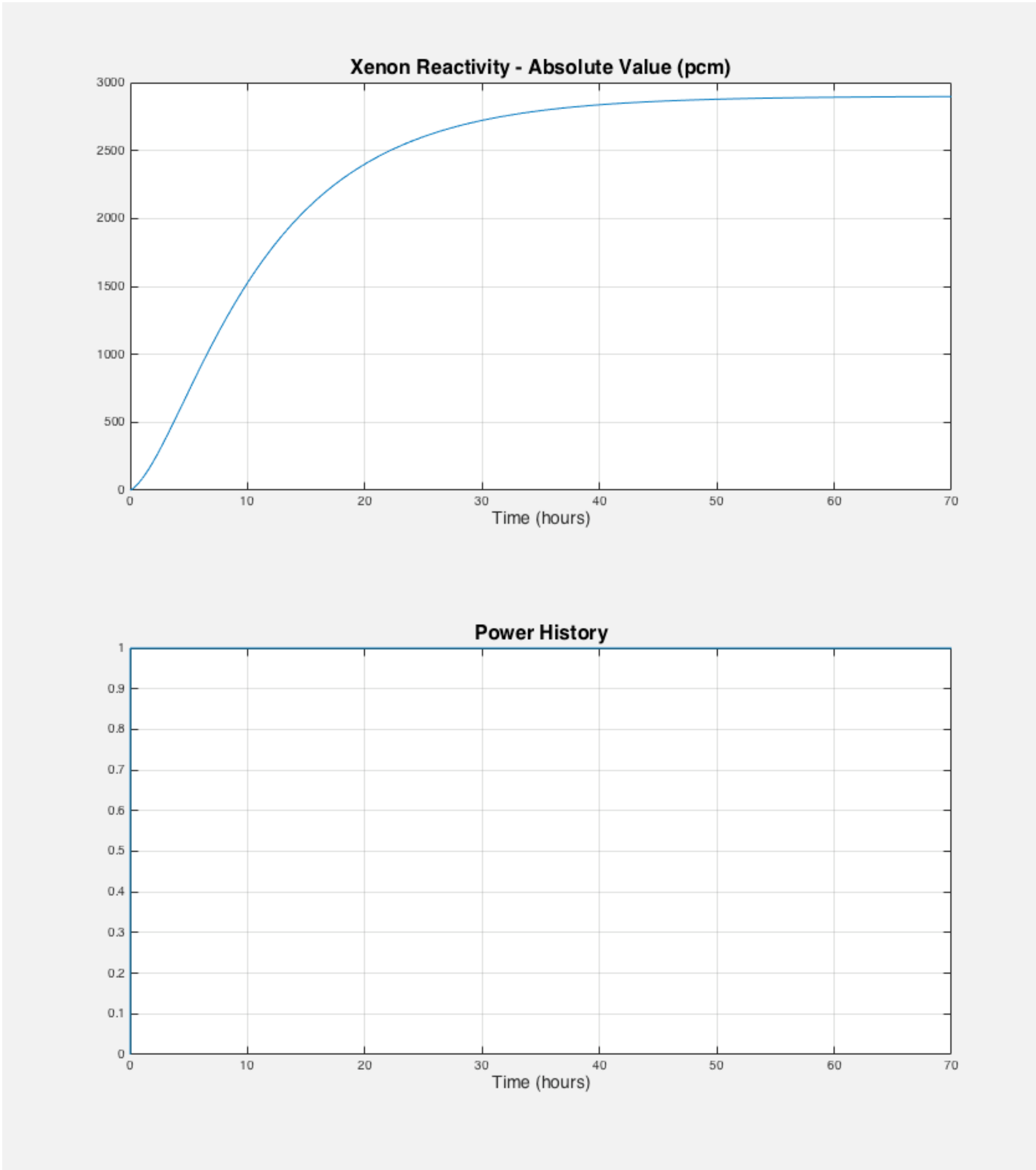
$\frac{X_k - X_{k-1}}{\tau_{step}^{[sec]}} = AX_k + B$	$X_k = G(X_{k-1} + B\tau_{step}^{[sec]})$ An only slightly more complicated solution
$G = (I - A\tau_{step}^{[sec]})^{-1}$	$G = \begin{bmatrix} \frac{1}{(1+\lambda_I\tau_{step}^{[sec]})} & 0 \\ \frac{\lambda_I\tau_{step}^{[sec]}}{(1+\tau_{step}^{[sec]}(\lambda_{Xe}+pR^{Max}))(1+\lambda_I\tau_{step}^{[sec]})} & \frac{1}{(1+\tau_{step}^{[sec]}(\lambda_{Xe}+pR^{Max}))} \end{bmatrix}$

The eigen values of A are not far apart. Hence this method works well even with the first order difference shown here. However, a slightly more complicated approach with a split difference works significantly better.

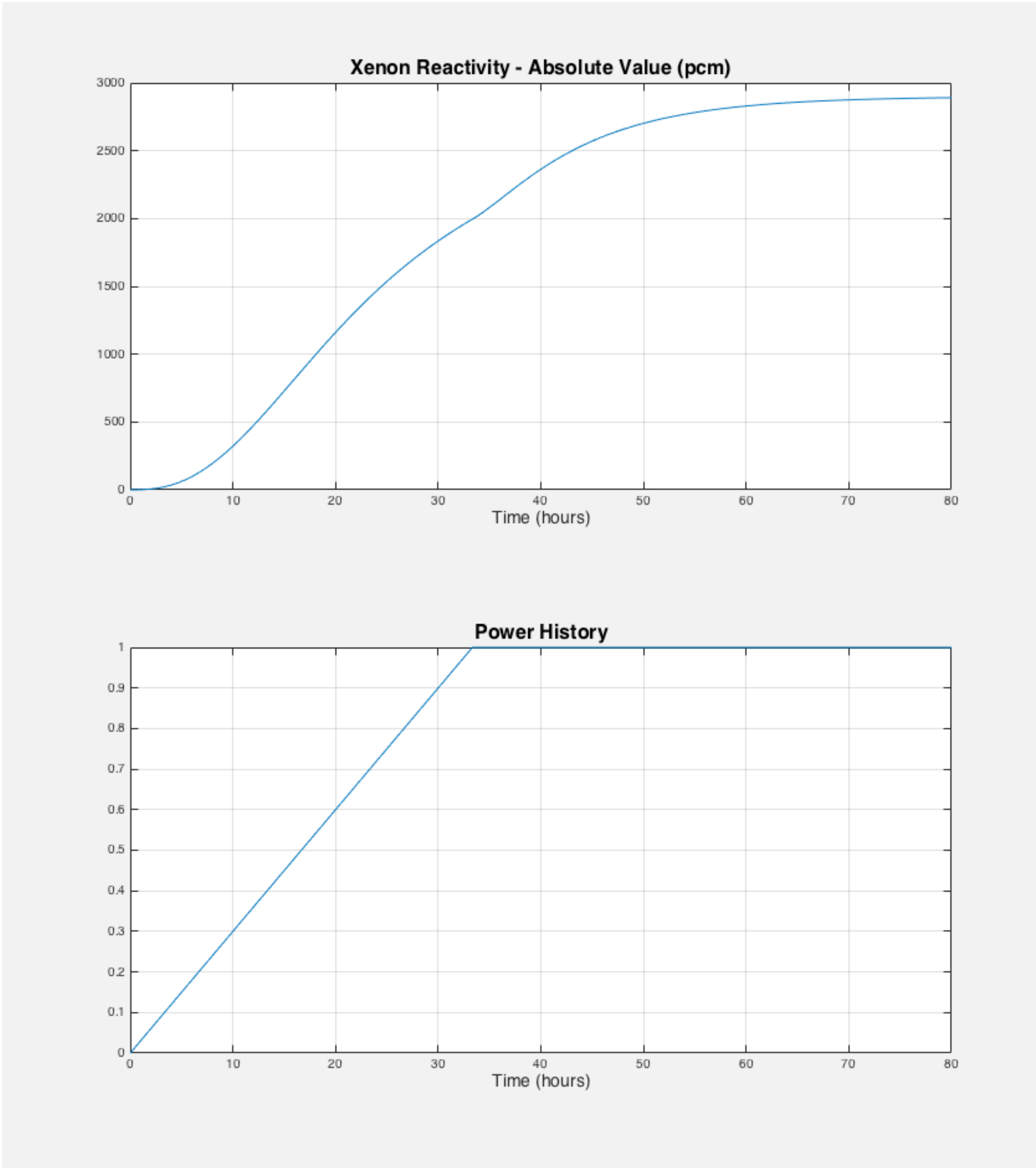
$\frac{X_k - X_{k-1}}{\tau_{step}^{[sec]}} = \frac{AX_k + AX_{k-1}}{2} + B$	$X_k = G(X_{k-1} + B\tau_{step}^{[sec]})$
$G = (I + \frac{A\tau_{step}^{[sec]}}{2})(I - \frac{A\tau_{step}^{[sec]}}{2})^{-1}$	D = $4*\lambda_I + 2(\lambda_{Xe} + pR^{Max}) + \lambda_I(\lambda_{Xe} + pR^{Max})\tau_{step}^{[sec]}$ $G = \frac{\begin{bmatrix} 2(\lambda_{Xe}+pR^{Max}) - \lambda_I(\lambda_{Xe}+pR^{Max})\tau_{step}^{[sec]} & 4\lambda_I \\ 4(\lambda_I\lambda_{Xe}+pR^{Max}) & (\lambda_{Xe}+pR^{Max})(2+\lambda_I\tau_{step}^{[sec]}) \end{bmatrix}}{D}$

The curves shown in this document use the first method using the first order difference.

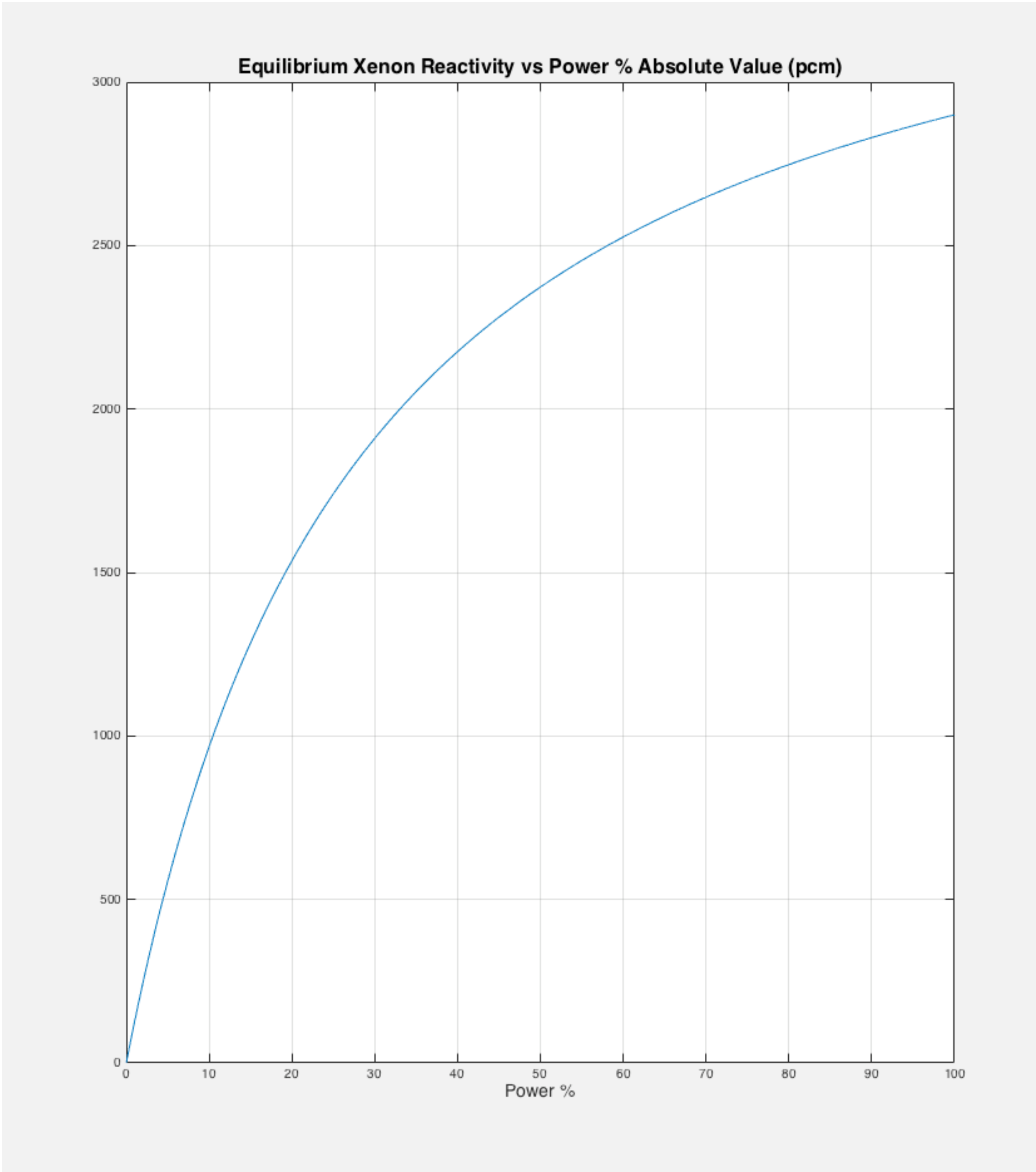
What follows are several example xenon transients to discuss.



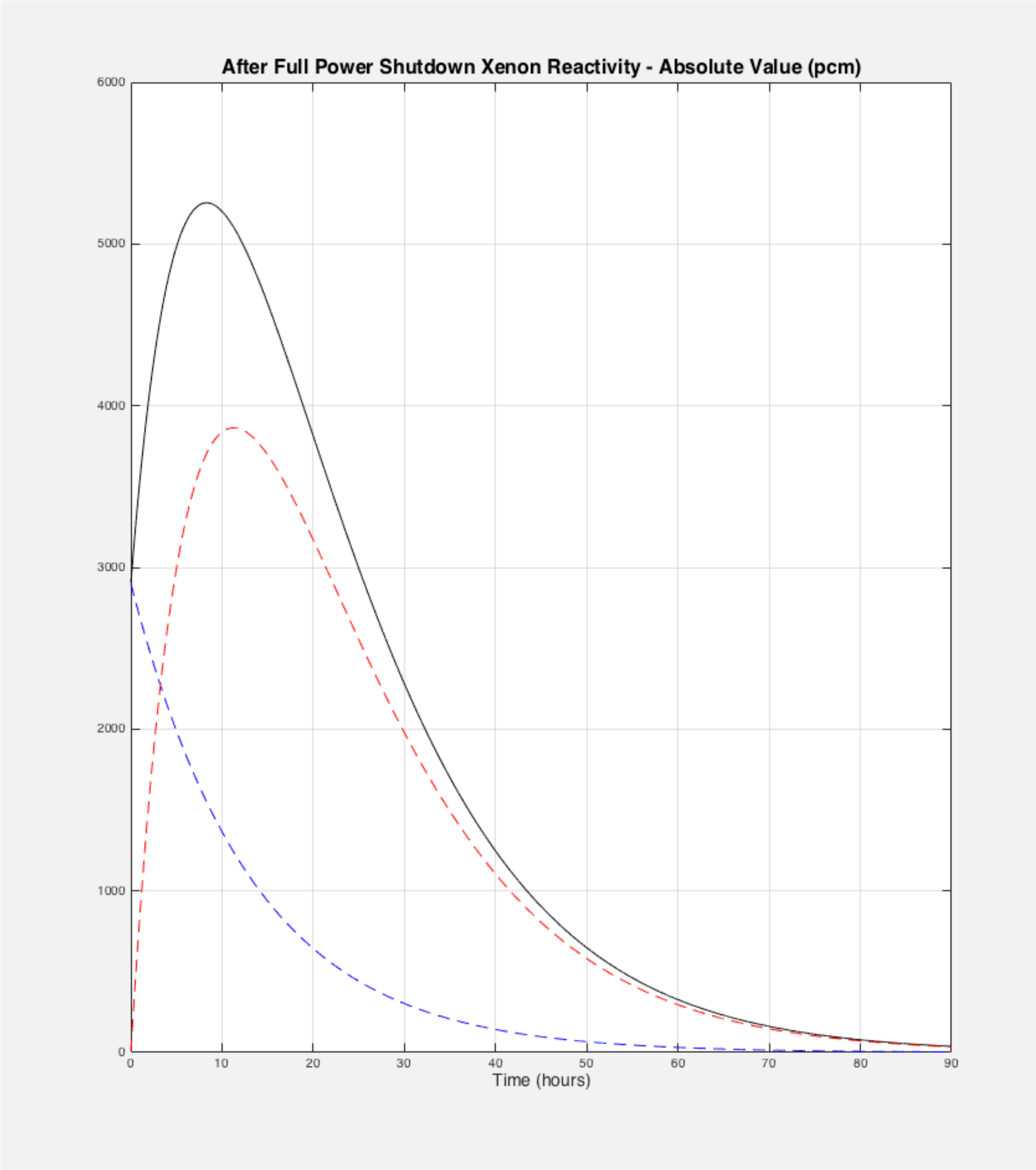
This figure demonstrates the buildup of xenon in the core that would happen following a step increase to 100% power. The final value in this case is 2900 pcm of negative reactivity.



This figure demonstrates the buildup of xenon in the core that would happen following a ramp increase to 100% power. The final value in this case is 2900 pcm of negative reactivity.

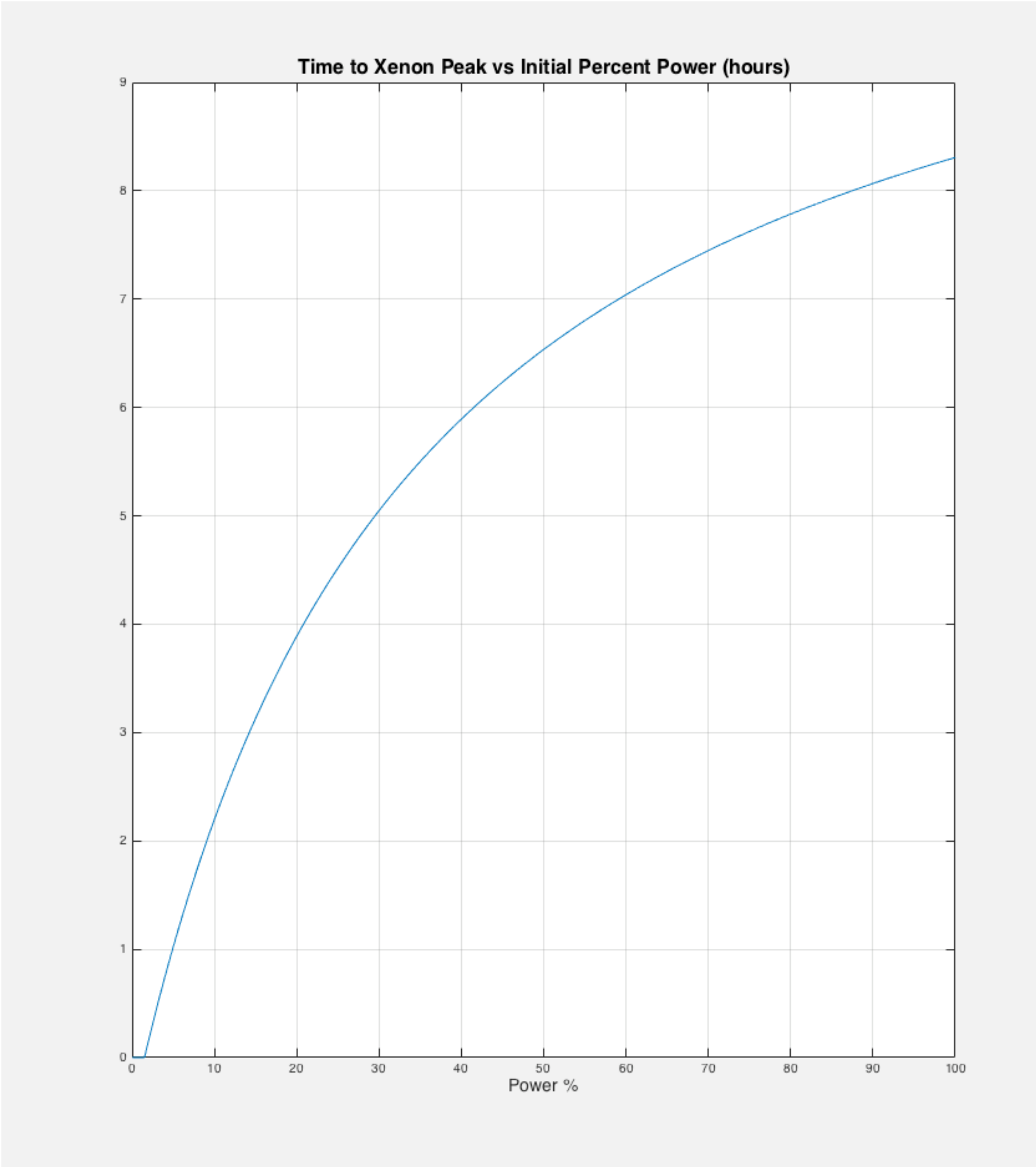


This figure demonstrates the behavior of equilibrium xenon as a function of power level. The strong rollover in this curve is due to the burnout of xenon. The production of xenon is proportional to power in equilibrium but the removal is also strongly dependent on power.



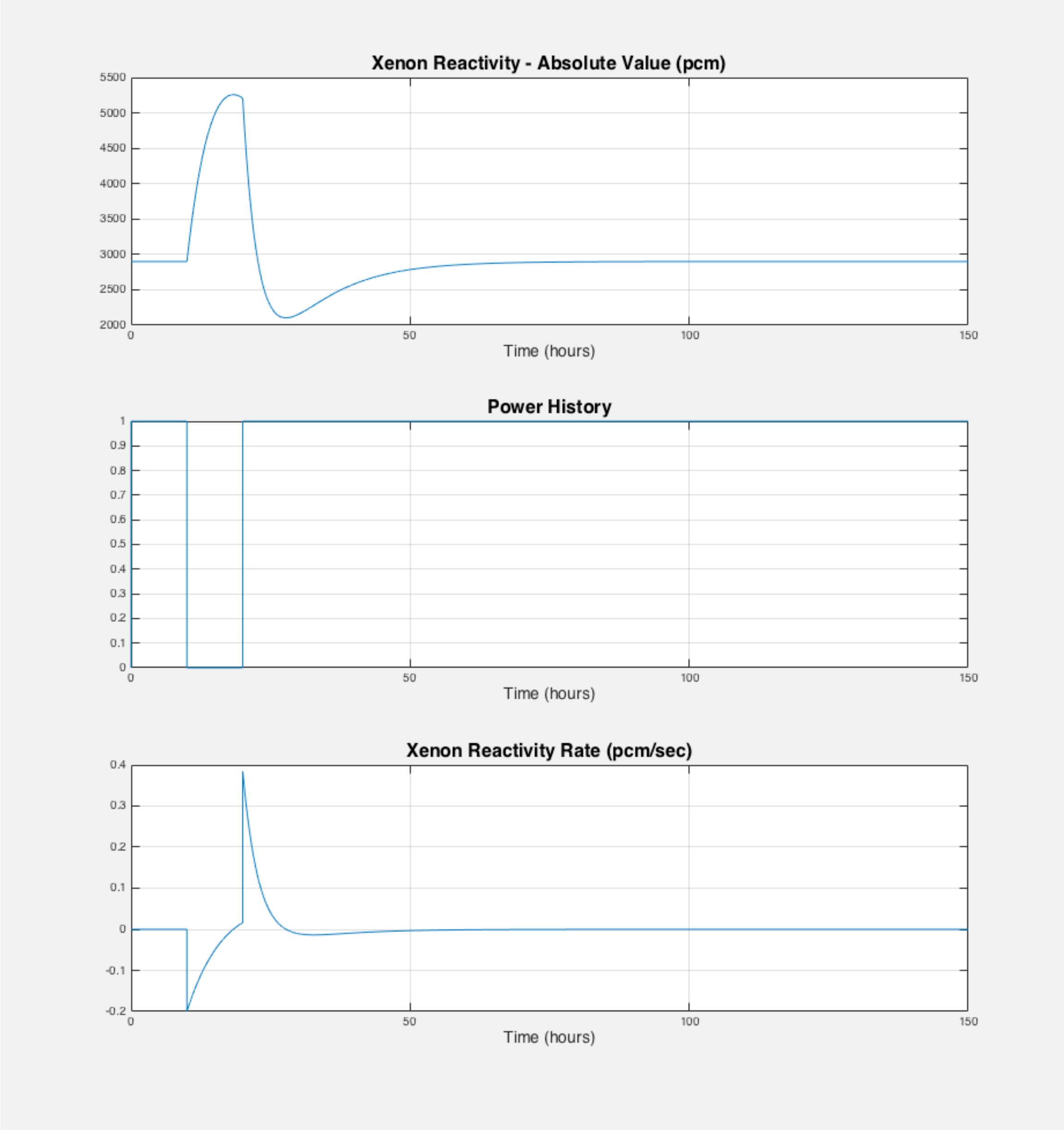
This plot demonstrates the makeup of peak xenon after shutdown. The dashed blue curve represents the decay of the xenon that was in the core at the time of the shutdown. This just decays with a 9.2 hour half-life. The dashed red line represents the contribution to the xenon from the iodine that existed in the core at the time of shutdown. This curve will peak at the same time independent of power (11.28 hours).

The magnitude of equilibrium iodine is proportional to power. The magnitude equilibrium xenon is less than a linear proportion due to the burn up of xenon. As a result as power rises, the sum is more strongly weighted to the iodine contribution moving the peak to a later time. In this case, full power, the peak is at 8.3 hours.

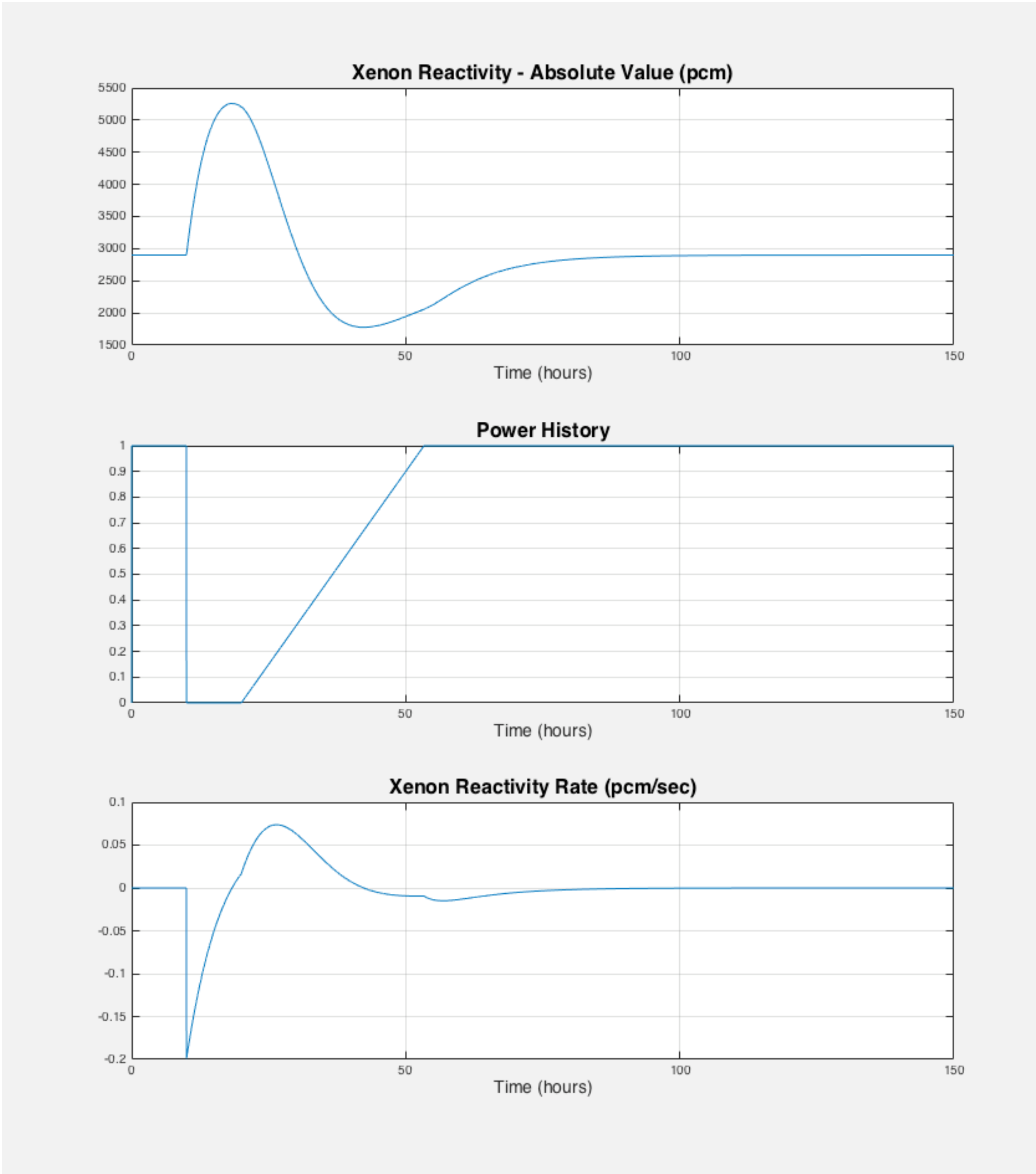


This shows the time to peak Xe after shutdown from a steady state power in percent. The time is in hours. This is a plot of the following equation. The quantity $p_{\min}^{peak}$ is the minimum power at which a peak will happen. This quantity is nearly 1.5% for the sample problem shown here.

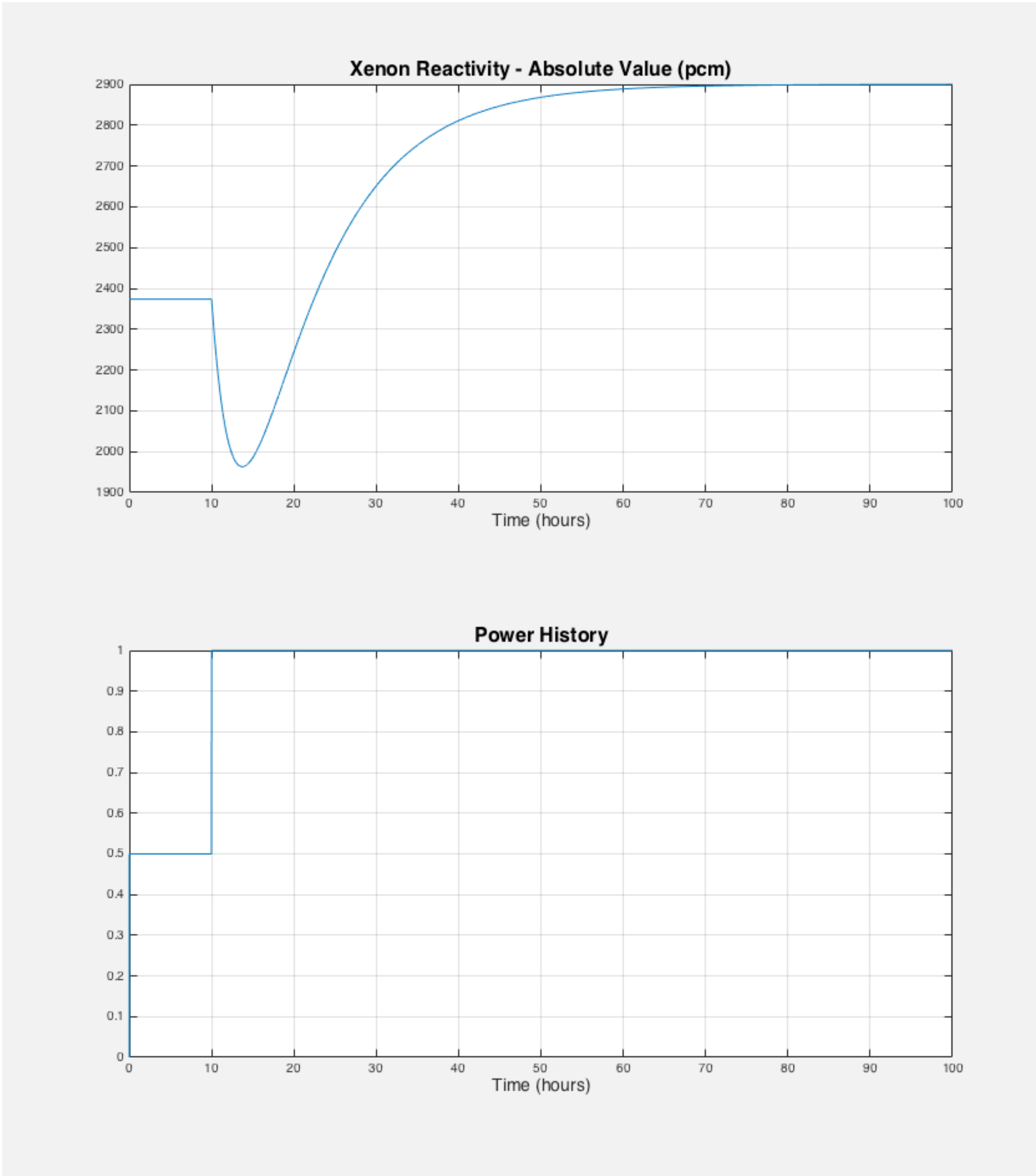
$$t_{peak} = \frac{1}{\lambda_I - \lambda_{Xe}} \ln\left(\frac{\frac{\lambda_I}{\lambda_{Xe}}}{1 + \frac{N_{Xe}^{Eq}}{N_I^{Eq}}\left(1 - \frac{\lambda_{Xe}}{\lambda_I}\right)}\right)$$	$$p_{min}^{peak} = \frac{\lambda_{Xe}\Upsilon_{Xe}}{\Upsilon_I R^{Max}}$$



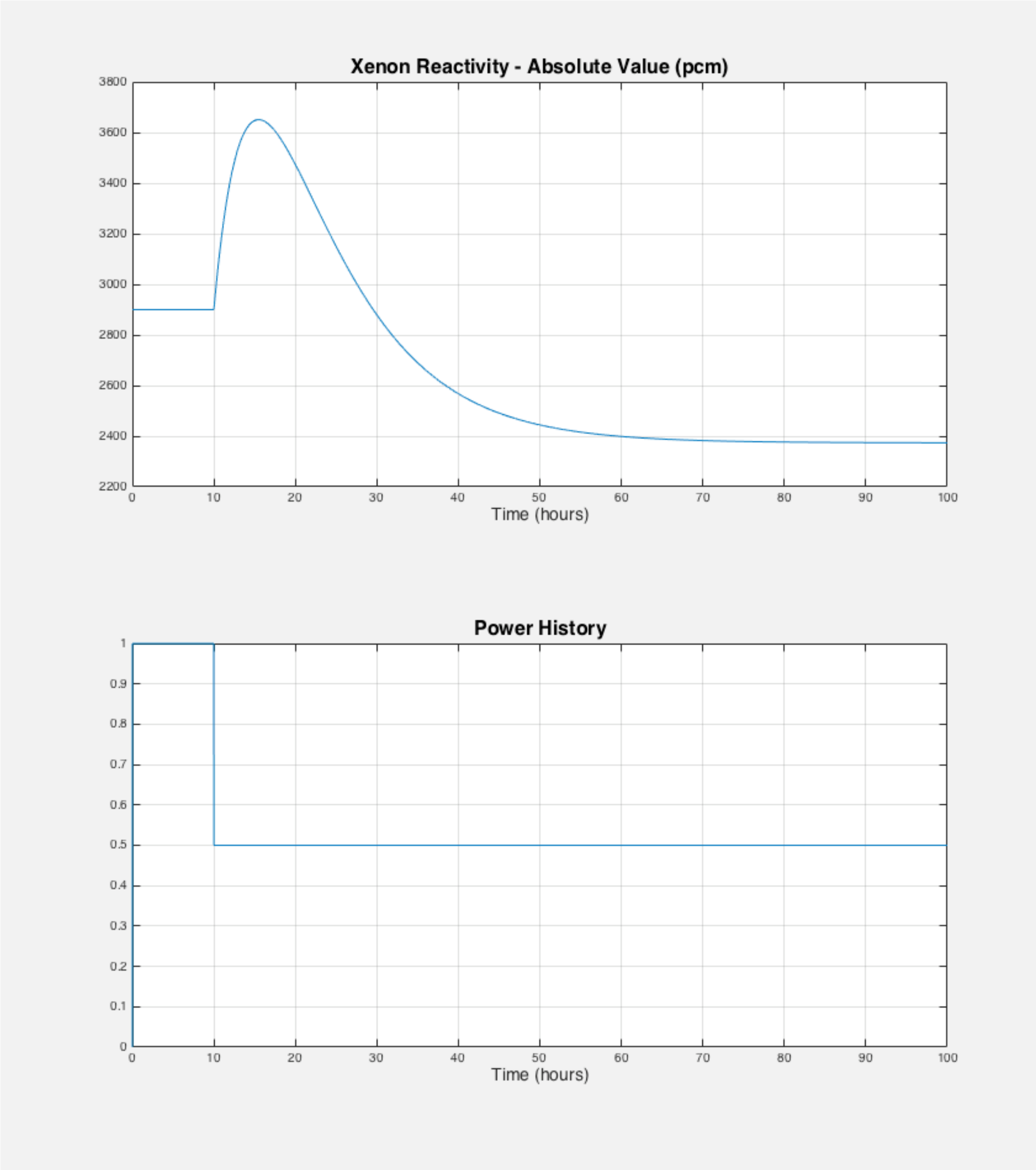
This is a sudden shutdown from full power followed by a step back up to 100% power at nearly the time of the peak xenon. The notable feature to this plot is the reactivity addition rate encountered just after the startup requiring rod motion or another reactivity control mechanism. The burn causes the xenon reactivity to drop from approximately 5300pcm to 2100 pcm in approximately 8 hours - with the first hour of that time accounting for a large part of the total change. The reactivity then recovers to the full power equilibrium value of 2900 pcm.



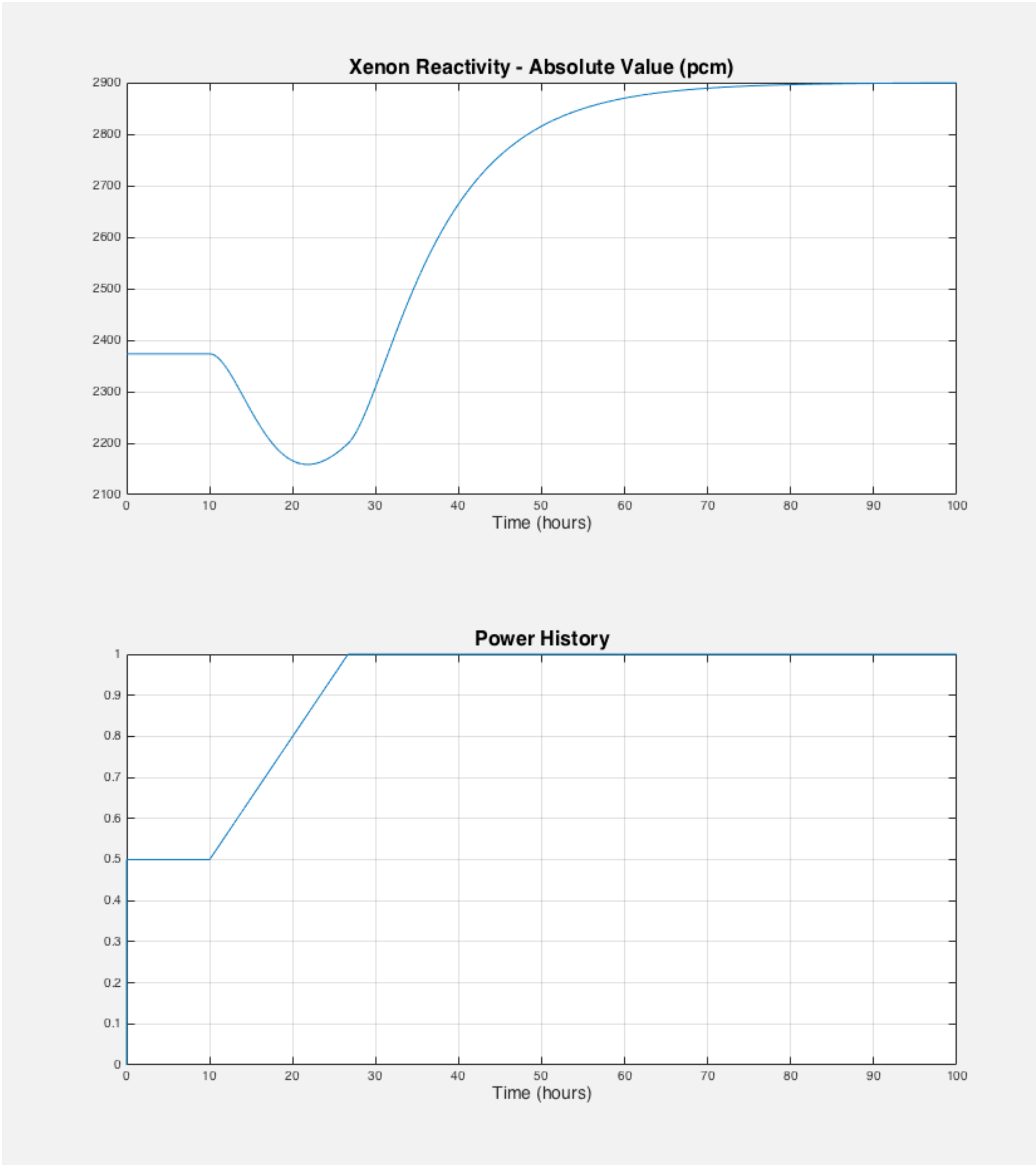
In this case a shutdown identical to the last figure is used and a startup with a ramp power increase of 3% per hour is used. Here the peak reactivity addition rate is significantly less than the previous example.



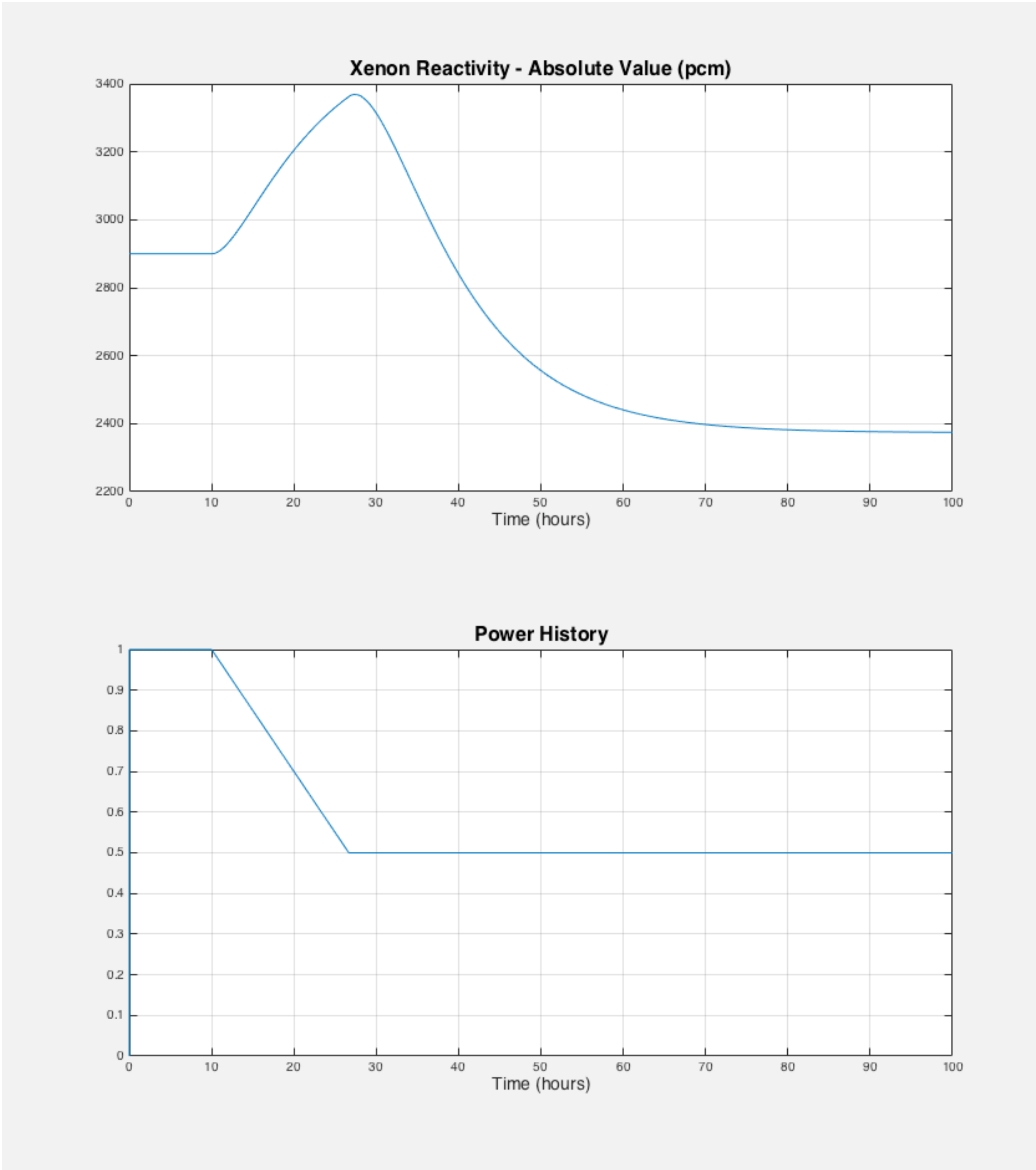
This figure demonstrates an up power transient step from 50% to 100% power. The initial dip is due to the sudden increase in the burn out taking place at the new higher power. In time however the xenon heads to its new equilibrium level for 100%. The dip turns in this case about 3.72 hours after the transient.



This step downward transient has a character similar to the xenon following a shutdown however it is less pronounced. This peak also happens at about 5.49 hours after the power transient.



This figure is similar to the up power transient shown in the previous figures. Here the power transient is at 3% per hour and the dip is much less pronounced. The bottom of the dip is shifted well to the right due to the slow rate of power increase.



This figure demonstrates a slow ramp reduction in power at 3% per hour. There is still a peak in xenon, but it is much smaller and later than in the step case.

Derive the Max Peak Xenon Equation:

First, we will find forms for the iodine and xenon concentrations as a function of time and then we will use the differential equation for xenon to derive the peak where the slope in the xenon is zero.

This solution could be easily derived using integration of the differential equations. As an alternative we will use the matrix exponential solution method for illustration.

Assume that the initial power (p) creates equilibrium xenon and iodine.
At shutdown p becomes zero and at the peak $\frac{\mathrm{dN_{Xe}}}{\mathrm{dt}} = 0$.

$\mathrm{A} = \begin{bmatrix} -\lambda_I & 0 \\ \lambda_I & -\lambda_{Xe} - pR^{Max} \end{bmatrix}$	$\mathrm{B} = \mathrm{pK}\begin{bmatrix} \gamma_I \\ \gamma_{Xe} \end{bmatrix}$	
$\frac{\mathrm{dX}}{\mathrm{dt}} = \mathrm{AX} + \mathrm{B}$	$\mathrm{X} = \begin{bmatrix} N_I \\ N_{Xe} \end{bmatrix}$	$X_0 = \begin{bmatrix} N_I^{Eq} \\ N_{Xe}^{Eq} \end{bmatrix}$

If p = 0, the differential equation reduces to:

$\frac{\mathrm{dX}}{\mathrm{dt}} = \mathrm{AX}$	$\mathrm{A} = \begin{bmatrix} -\lambda_I & 0 \\ \lambda_I & -\lambda_{Xe} \end{bmatrix}$

The eigen value diagonal and eigen vector matrixes are:

$\mathrm{E} = \begin{bmatrix} -\lambda_I & 0 \\ 0 & -\lambda_{Xe} \end{bmatrix}$	$\mathrm{M} = \begin{bmatrix} -\frac{\lambda_I - \lambda_{Xe}}{\lambda_I} & 0 \\ 1 & 1 \end{bmatrix}$	$\mathrm{M}^{-1} = \begin{bmatrix} \frac{-\lambda_I}{\lambda_I - \lambda_{Xe}} & 0 \\ \frac{\lambda_I}{\lambda_I - \lambda_{Xe}} & 1 \end{bmatrix}$

We obtain e^{At} using the eigenvalue diagonal matrix and the eigenvector (modal) matrix as we did in the Fundamental Kinetics Ideas handout.

$$e^{At} = \mathrm{M}^{-1}\begin{bmatrix} e^{-\lambda_I t} & 0 \\ 0 & e^{-\lambda_{Xe} t} \end{bmatrix} M = \begin{bmatrix} e^{-\lambda_I t} & 0 \\ \frac{\lambda_I (e^{-\lambda_{Xe} t} - e^{-\lambda_I t})}{\lambda_I - \lambda_{Xe}} & e^{-\lambda_{Xe} t} \end{bmatrix}$$

And $\mathrm{X(t)} = e^{At} X_0$.

$$X(t) = \begin{bmatrix} N_I^{Eq} e^{-\lambda_I t} \\ N_I^{Eq} \frac{\lambda_I (e^{-\lambda_{Xe} t} - e^{-\lambda_I t})}{\lambda_I - \lambda_{Xe}} + N_{Xe}^{Eq} e^{-\lambda_{Xe} t} \end{bmatrix}$$

We know that the peak in xenon happens when $\frac{d\mathrm{N_{Xe}}}{dt} = 0$ and from the second row of A we see that this condition will happen when $[\,\lambda_I, -\lambda_{\mathrm{Xe}}] * X$ is zero. We need to find the time when the following equation holds:

$$\lambda_I(N_I^{Eq}e^{-\lambda_I t}) - \lambda_{\mathrm{Xe}}\left(N_I^{Eq}\frac{\lambda_I\left(e^{-\lambda_{\mathrm{Xe}}t} - e^{-\lambda_{\mathrm{I}}t}\right)}{\lambda_I - \lambda_{\mathrm{Xe}}} + N_{Xe}^{Eq}e^{-\lambda_{\mathrm{Xe}}t}\right) = 0$$

Multiply by $(\lambda_I - \lambda_{\mathrm{Xe}})e^{\lambda_I t}/(\lambda_I\lambda_{\mathrm{Xe}}N_I^{Eq})$:

$$\left(\frac{\lambda_I}{\lambda_{\mathrm{Xe}}} - 1\right) - \left(\left(e^{(\lambda_I - \lambda_{\mathrm{Xe}})t} - 1\right) + \frac{N_{Xe}^{Eq}}{N_I^{Eq}}(1 - \lambda_{\mathrm{Xe}}/\lambda_I)e^{(\lambda_I - \lambda_{\mathrm{Xe}})t}\right) = 0$$

Now collect the exponential terms:

$$\left(1 + \frac{N_{Xe}^{Eq}}{N_I^{Eq}}(1 - \lambda_{\mathrm{Xe}}/\lambda_I)\right)e^{(\lambda_I - \lambda_{\mathrm{Xe}})t} = \frac{\lambda_I}{\lambda_{\mathrm{Xe}}}$$

So

$$t_{peak} = \frac{1}{\lambda_{\mathrm{I}} - \lambda_{Xe}}\ln\left(\frac{\frac{\lambda_{\mathrm{I}}}{\lambda_{Xe}}}{1 + \frac{N_{Xe}^{Eq}}{N_I^{Eq}}(1 - \frac{\lambda_{Xe}}{\lambda_I})}\right)$$

The criteria for a peak after shutdown is this this value be greater than zero. This is to say that $\lambda_{\mathrm{I}}N_I^{Eq} - \lambda_{Xe}N_I^{Eq}$>0. At the limit the two quantities would be one. This happens if:

$$\frac{\lambda_{Xe}N_I^{Eq}}{\lambda_{\mathrm{I}}N_I^{Eq}} = 1$$

$N_I^{Eq} = \frac{\gamma_{\mathrm{I}}Kp}{\lambda_{\mathrm{I}}}$	$N_{Xe}^{Eq} = \frac{(\gamma_{\mathrm{I}} + \gamma_{\mathrm{Xe}})Kp}{\lambda_{\mathrm{Xe}} + \mathrm{pR^{Max}}}$

Using the equations derived at the start of this paper for these values the ratio becomes:

$$\frac{\lambda_{Xe}N_{Xe}^{Eq}}{\lambda_{\mathrm{I}}N_I^{Eq}} = \frac{\frac{\lambda_{Xe}(\gamma_{\mathrm{I}} + \gamma_{\mathrm{Xe}})Kp}{\lambda_{\mathrm{Xe}} + \mathrm{pR^{Max}}}}{\lambda_{\mathrm{I}}\frac{\gamma_{\mathrm{I}}Kp}{\lambda_{\mathrm{I}}}} = \frac{\lambda_{Xe}(\gamma_{\mathrm{I}} + \gamma_{\mathrm{Xe}})/\gamma_{\mathrm{I}}}{\lambda_{\mathrm{Xe}} + \mathrm{pR^{Max}}} = 1$$

Solving this for p gives:

$$p_{min}^{peak} = \frac{1}{\mathrm{R^{Max}}}\,\left(\frac{\lambda_{Xe}(\gamma_{\mathrm{I}} + \gamma_{\mathrm{Xe}})}{\gamma_{\mathrm{I}}}\right) - \lambda_{\mathrm{Xe}} = \frac{\gamma_{\mathrm{Xe}}\lambda_{Xe}}{\mathrm{R^{Max}}\gamma_{\mathrm{I}}}$$

Constant Power Direct Solution

When the power is constant the xenon-iodine equations may be directly solved as follows:

Xenon and Iodine differential equation

$\mathrm{A} = \begin{bmatrix} -\lambda_{\mathrm{I}} & 0 \\ \lambda_{\mathrm{I}} & -\lambda_{\mathrm{Xe}} - \mathrm{pR}^{\mathrm{Max}} \end{bmatrix}$	$\mathrm{B} = \mathrm{pK}\begin{bmatrix} \gamma_{\mathrm{I}} \\ \gamma_{\mathrm{Xe}} \end{bmatrix}$
$\frac{\mathrm{dX}}{\mathrm{dt}} = \mathrm{AX} + \mathrm{B}$	$\mathrm{X} = \begin{bmatrix} \mathrm{N_I} \\ \mathrm{N_{Xe}} \end{bmatrix}$

Equilibrium values are as follows

$X^{Eq} = -A^{-1}B$	$A^{-1} = \begin{bmatrix} -\lambda_{\mathrm{Xe}} - \mathrm{pR}^{\mathrm{Max}} & 0 \\ -\lambda_{\mathrm{I}} & -\lambda_{\mathrm{I}} \end{bmatrix} * \frac{1}{\lambda_{\mathrm{I}}(\lambda_{\mathrm{Xe}} + \mathrm{pR}^{\mathrm{Max}})}$
$N_I^{Eq} = \frac{\gamma_{\mathrm{I}} Kp}{\lambda_{\mathrm{I}}}$	$N_{Xe}^{Eq} = \frac{(\gamma_{\mathrm{I}} + \gamma_{\mathrm{Xe}})Kp}{\lambda_{\mathrm{Xe}} + \mathrm{pR}^{\mathrm{Max}}}$

Using the integrating factor e^{-At}, the differential equation may be written as:

$$\frac{\mathrm{d}(e^{-At}\mathrm{X})}{\mathrm{dt}} = e^{-At}\mathrm{B}$$

So:

$$X(t) = e^{At}\mathrm{X}(0) + e^{At}\int_{t_0}^{t} e^{-At'}\mathrm{B}\,\mathrm{d}t'$$

And integrating we get:

$$X(t) = e^{At}\mathrm{X}(0) + e^{At}\int_{t_0}^{t} e^{-At'}\mathrm{B}\,\mathrm{d}t'$$

And this becomes:

$$X(t) = e^{At}\mathrm{X}(0) - A^{-1}(I - e^{At})B$$

With the substitutions:

$N_I(t) = N_I^0 e^{-\lambda_{\mathrm{I}} t} + K\gamma_{\mathrm{I}} p(1 - e^{-\lambda_{\mathrm{I}} t})/\,\lambda_{\mathrm{I}}$
$N_{Xe}(t) = N_{Xe}^0 e^{-(\lambda_{\mathrm{Xe}}+\mathrm{pR}^{\mathrm{Max}})t} + \lambda_{\mathrm{I}} N_I^0 (e^{-(\lambda_{\mathrm{Xe}}+\mathrm{pR}^{\mathrm{Max}})t} - e^{-\lambda_{\mathrm{I}} t})/(\lambda_{\mathrm{I}} - \lambda_{\mathrm{Xe}} - \mathrm{pR}^{\mathrm{Max}}) + K\gamma_{\mathrm{Xe}} p(1 - e^{-(\lambda_{\mathrm{Xe}}+\mathrm{pR}^{\mathrm{Max}})t})/(\lambda_{\mathrm{Xe}} + \mathrm{pR}^{\mathrm{Max}}) + K\gamma_{\mathrm{I}} p(\lambda_{\mathrm{I}}(1 - e^{-(\lambda_{\mathrm{Xe}}+\mathrm{pR}^{\mathrm{Max}})t}) - (\lambda_{\mathrm{Xe}} + \mathrm{pR}^{\mathrm{Max}})(1 - e^{-\lambda_{\mathrm{I}} t})\,))/((\lambda_{\mathrm{Xe}} + \mathrm{pR}^{\mathrm{Max}})(\lambda_{\mathrm{I}} - \lambda_{\mathrm{Xe}} - \mathrm{pR}^{\mathrm{Max}}))$

It is important to note that the form showing X(t) can be computed directly without resorting to the much more complicated for the components shown above. This is a clear example of the benefit of thinking directly in terms of matrix arithmetic.