



University of Pittsburgh

ME/ENGR 2100

Fundamentals of Nuclear Engineering

Fission Reactor Basics

Dr. Daniel F. Gill

Stephen R. Tritch Program in Nuclear Engineering
Swanson School of Engineering
University of Pittsburgh





Relevant Reading Assignments

- Sections 6.5 to 6.8 of “*Introduction to Nuclear Engineering*” by Lamarsh and Baratta, 3rd Edition.
- Chapter 3 of “*Nuclear Reactor Analysis*” by Duderstadt and Hamilton
- Page 100-120 of “*Nuclear Engineering: Theory and Technology of Commercial Nuclear Power*” by Knief, 2nd Edition.



Relevant Reading Assignments

- “Secrecy, simultaneous discovery, and the theory of nuclear reactors” by Spencer Weart. American Journal of Physics, Vol. 45(11). November 1977



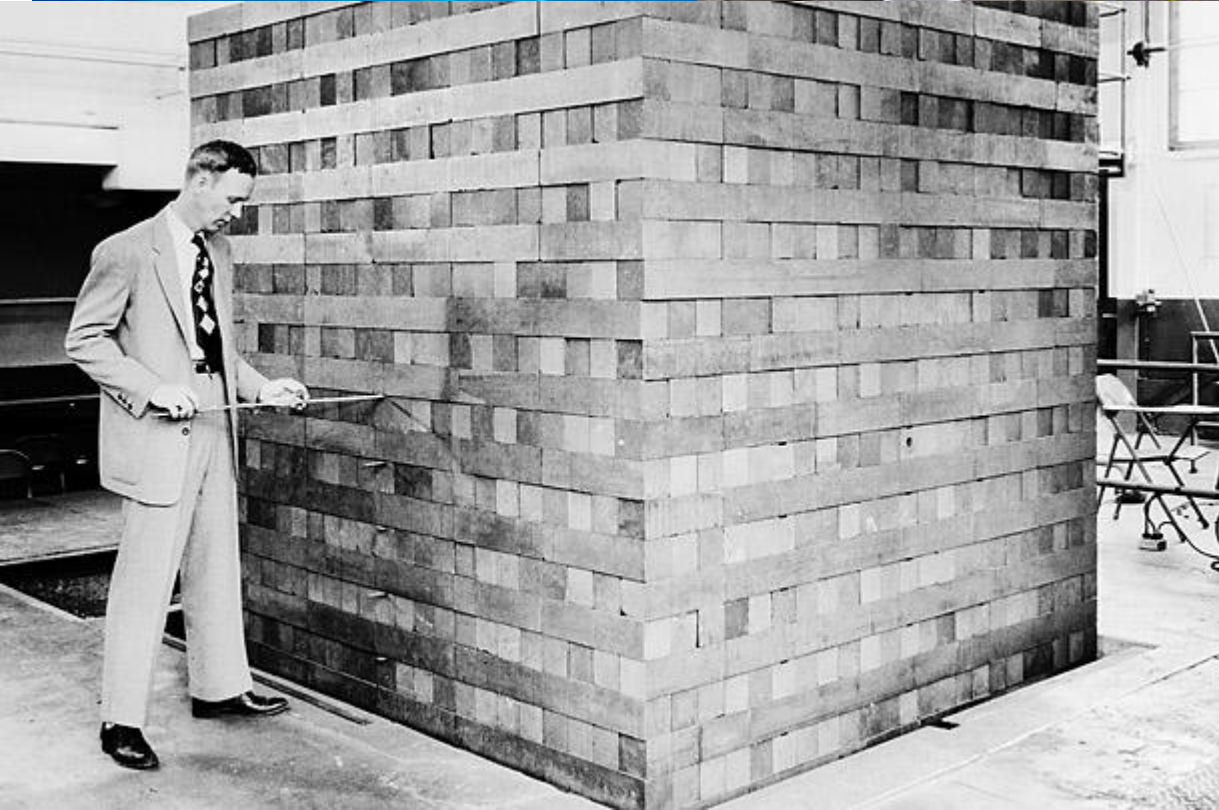
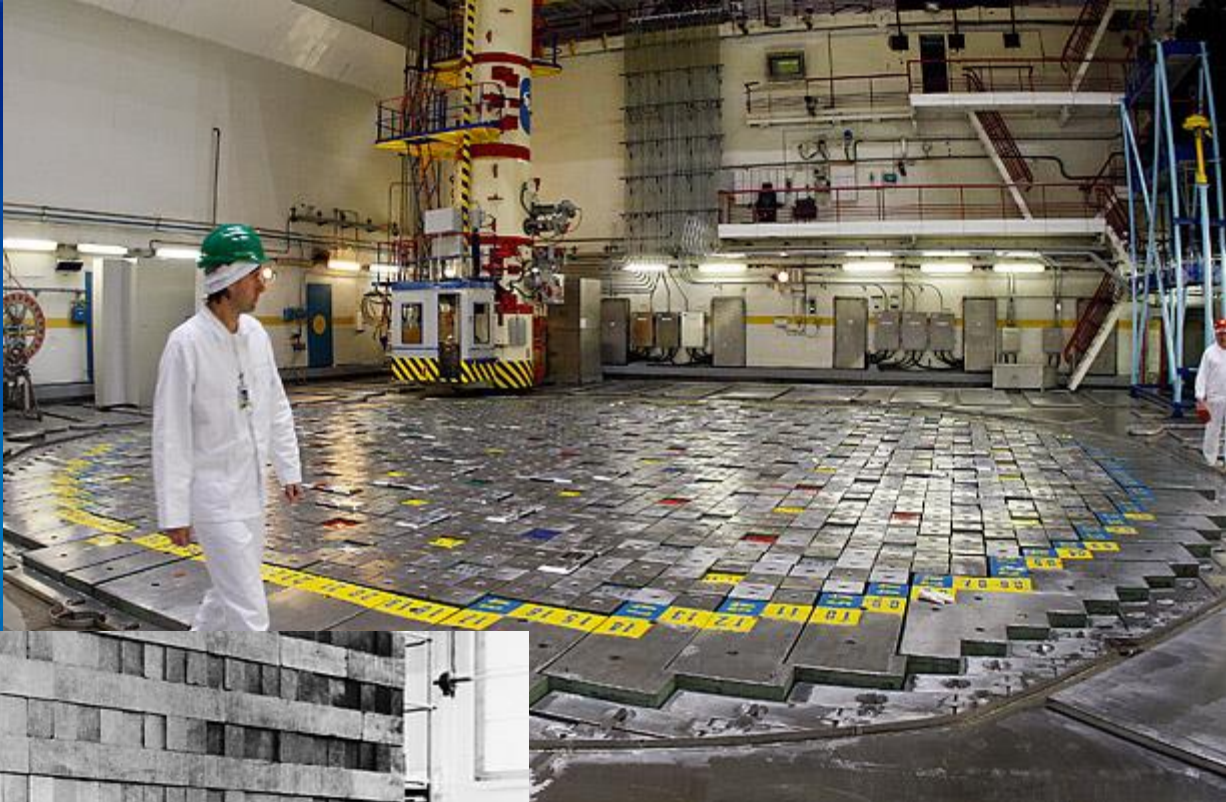
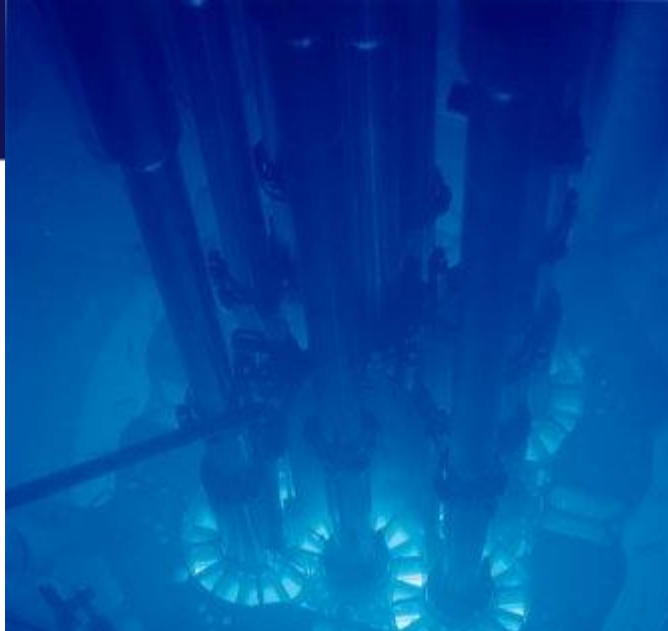
Learning Objectives

- Describe the identifying characteristics of a nuclear reactor
- Differentiate among critical, supercritical, and subcritical conditions in a reactor
- Identify the terms in the four and six factor formulas
- Explain the principle of neutron moderation by light nuclei and the importance to thermal reactors
- Understand the impact of heterogeneity on neutron balance
- Describe PWR and BWR fuel assemblies and some of the differences between them
- Differentiate between the infinite (k_{∞}) and effective (k_{eff} or k) multiplication factors
- Explain how the terms in the four and six factor formulas may be adjusted to control criticality in reactor and processing settings



Classifying Reactors

- By type of reaction
 - Fusion
 - Fission (fast neutrons, thermal neutrons)
- By moderator material (thermal reactors only)
 - Graphite
 - Water (heavy and light)
 - Light elements (Be, Li)
- By coolant
 - Water (PWR, Heavy Water PWR, BWR, Pool)
 - Liquid metal
 - Gas cooled
 - Molten salt
- By generation (I-IV)
- By use
 - Electricity (nuclear power plant)
 - Propulsion (marine propulsion)
 - Heat (desalination, domestic/industrial heating, hydrogen production)
 - Transmutation (breeding fuel, isotope production, weapons-grade material production)
 - Research (training, materials testing)



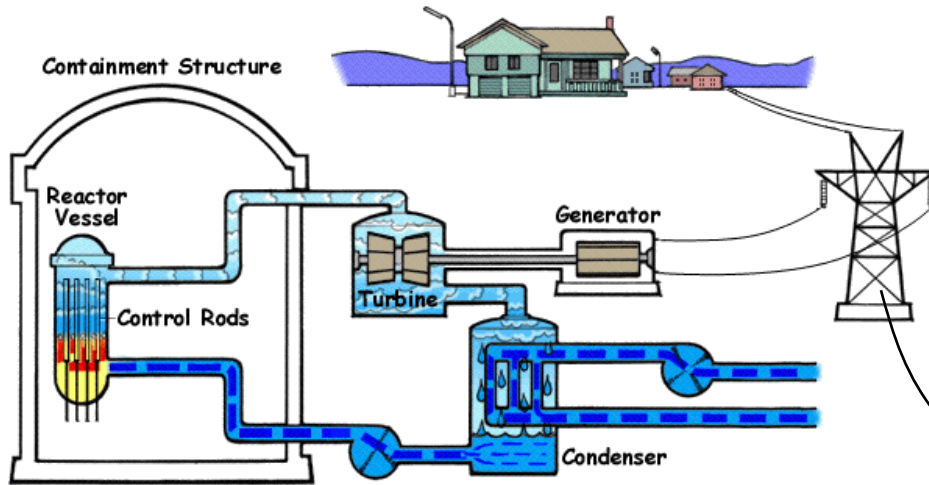


Reactor Properties

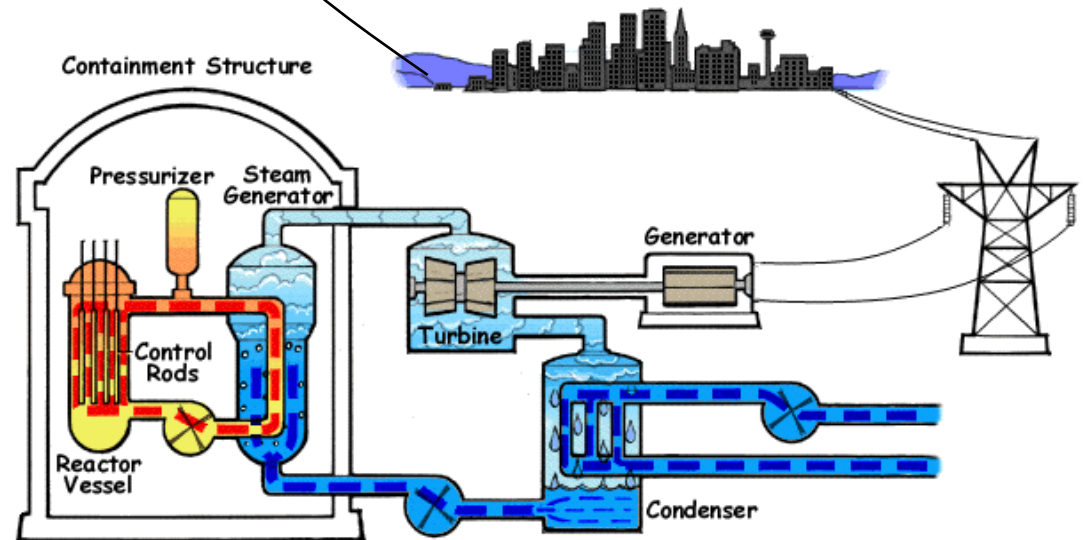
- Many types of reactors exist but they all share some important requirements
 - Nuclear fuel (fissile + fissionable)
 - Reactivity control mechanisms
 - Cooling capability
- Heat created through fission
 - Kinetic energy of fission products
 - Absorption of fission photons
 - Decay heat (radioactive decay)

Nuclear Power Plants

Nuclear Plant (BWR)



Nuclear Plant (PWR)



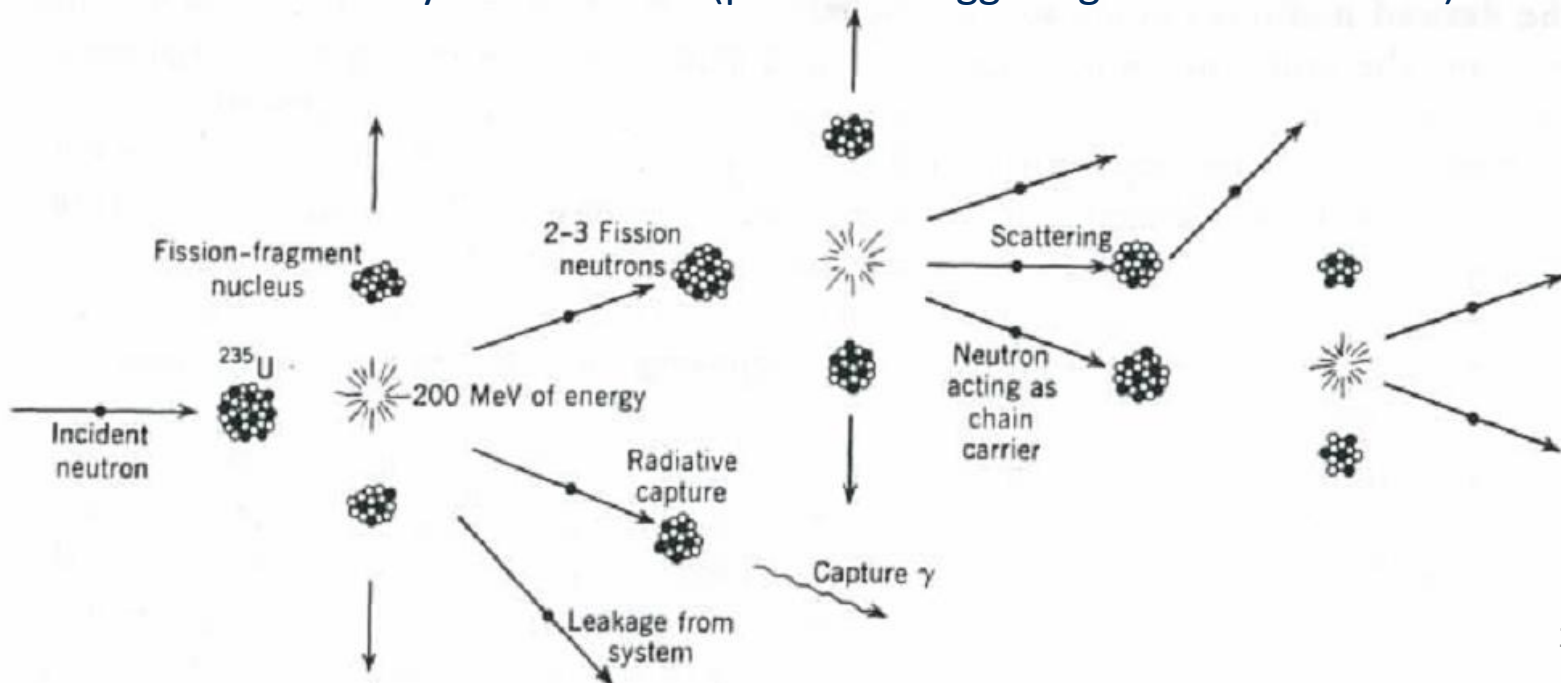


Neutron Economy

- Nuclear reactor core design focuses on the neutron economy within a reactor during its operating lifetime
- A successful reactor design must
 - Produce enough excess neutrons to keep the chain reaction going
 - Limit the number of excess neutrons so that the reaction does not become uncontrolled
 - Consider thermal and material limits as well!
- Nuclear designers balance neutron sources (fuel) with neutron absorbers and leakage, the rate at which neutrons escape from the core.

Neutron Life-Cycle

- **Birth:** Neutrons are born during fission events
- **Lifetime:** The lifetime of the neutron is the time between its birth and death. During this time the neutron potentially undergoes many scattering reactions off of host nuclei in the system
- **Death:** Neutron death occurs when the neutron leaks from the system or is absorbed by a host nuclei (potential triggering a fission reaction).



Life-Cycle Definition of k

- Accumulation boiled down to single number by defining **multiplication factor, k**

$$k = \frac{\text{Number of neutrons in one generation}}{\text{Number of neutrons in preceding generation}}$$

- Measures how many (average) neutrons are produced by each neutron born
- Characterizes the chain-reaction
 - Each neutron born must itself create at least 1 more neutron before being absorbed/leaking to sustain reaction



Criticality

- **Critical:** Reactor is static. The number of neutrons from generation to generation does not change
- **Subcritical:** Number of neutrons from generation to generation decreases, the reaction eventually dies out
- **Supercritical:** Number of neutrons from generation to generation increases without bound

$k < 1$ subcritical

$k = 1$ critical

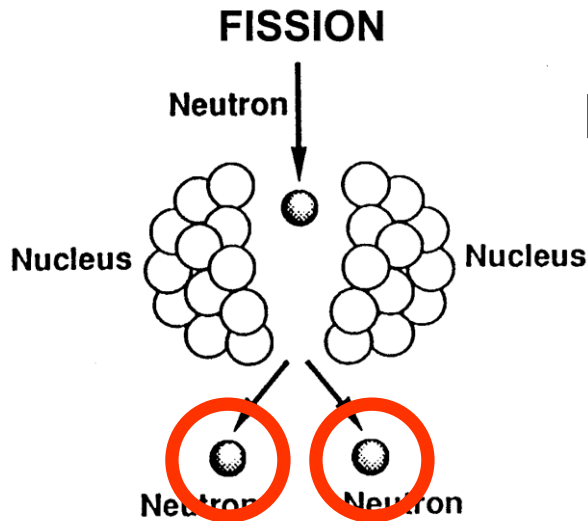
$k > 1$ supercritical



Determining k

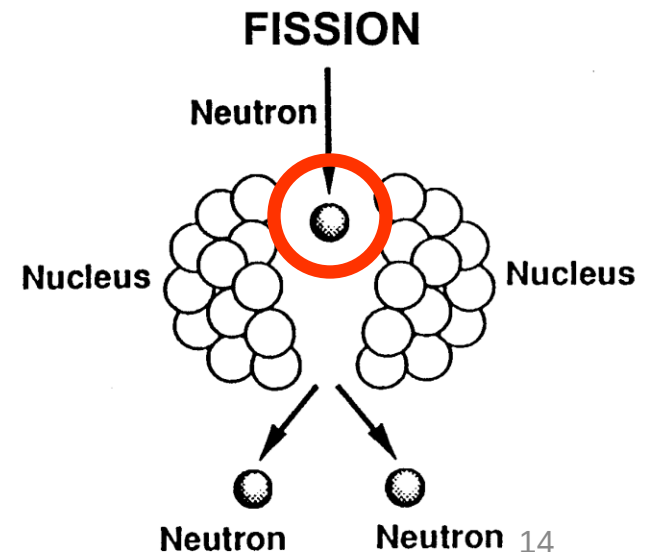
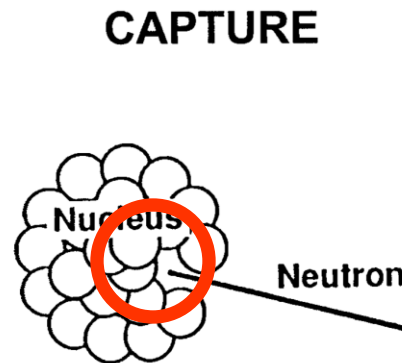
- Finding k is crucial in reactor design
- Today the determination of k is done using mathematical theories and computer hardware not available to the first reactor designers
- Original theories primarily based on physical intuition and written in terms of experimentally measurable quantities
- These theories distinguish between infinite (easier to quantify) and finite systems (practical)

Nuclear Reactions



Production = Fission Rate ($\Sigma_f \Phi$)
 $\times$ Neutrons produced
 per fission (ν)

Destruction Rate = Absorption Rate ($\Sigma_a \Phi$)



Infinite System

- Neutron Balance
 - Production Rate $\leftrightarrow$ Absorption Rate

$$\nu \Sigma_f \Phi \quad \Sigma_a \Phi$$

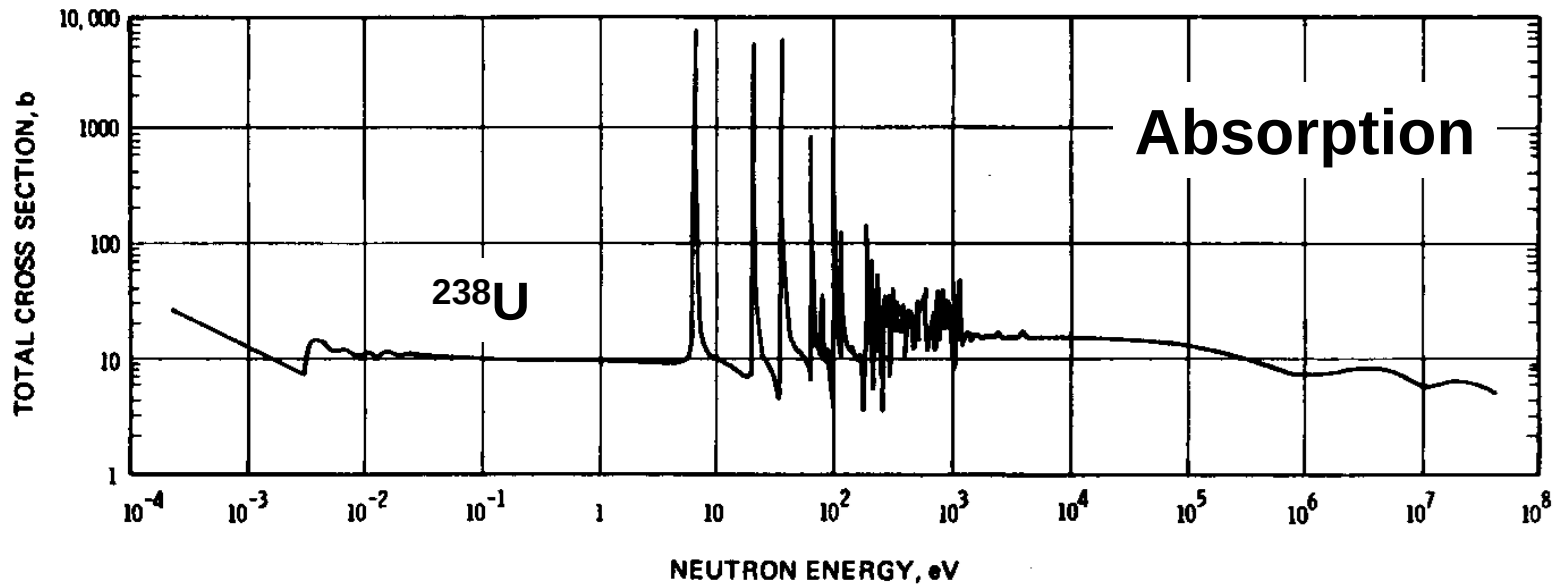
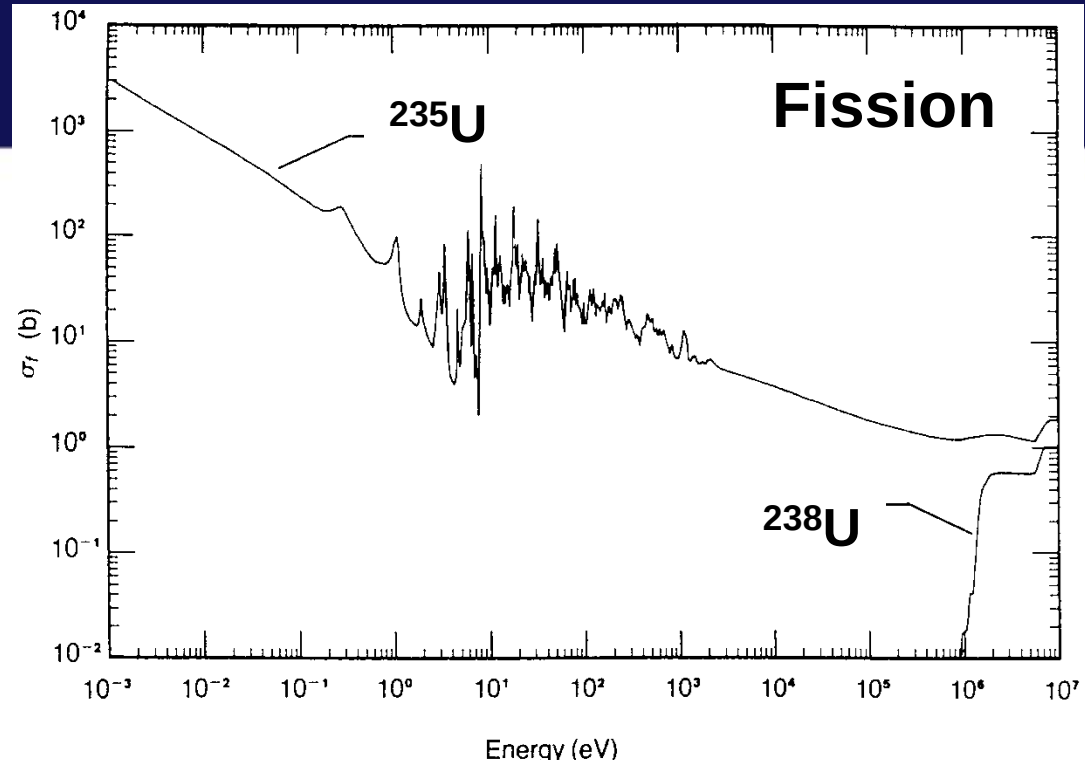
- (Infinite) Multiplication Factor

$$k_{\infty} = \frac{\text{Production Rate}}{\text{Absorption Rate}} = \frac{\nu \Sigma_f}{\Sigma_a}$$

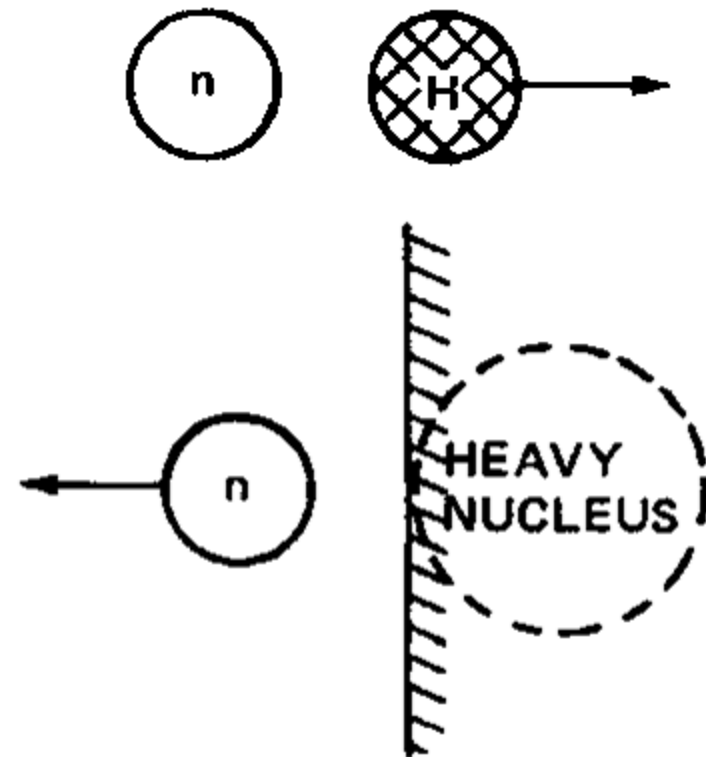
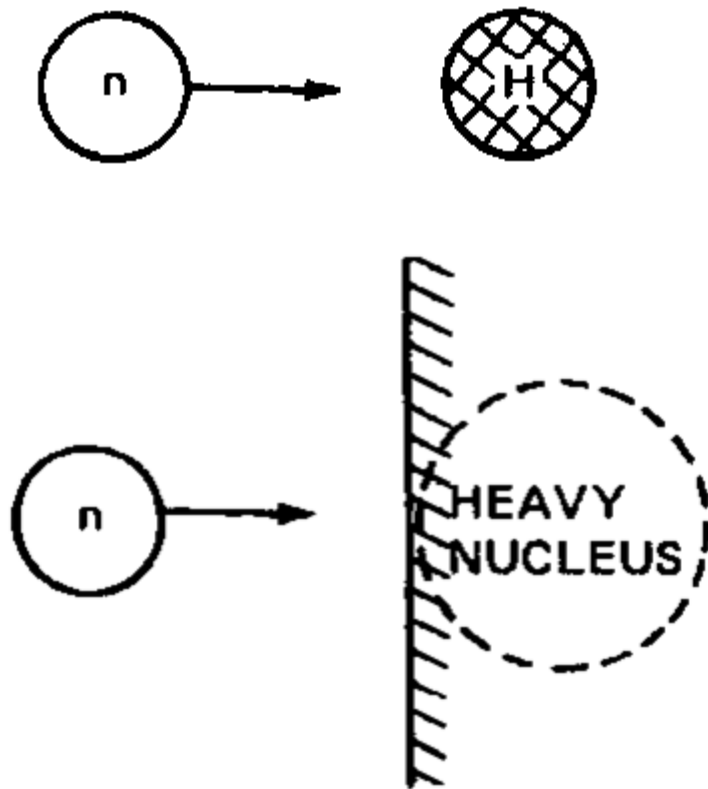
- Simplified model using one energy group
- Everything has been effectively energy averaged



Cross Section (Energy Dependence)



Neutron Scattering





Neutron Moderator Materials

Neutron Moderation Properties of Selected Atoms and Molecules

Moderator Target	Atomic Mass (A)	Scattering Ratio (α)	(1- α)	Collisions to Thermal [†]
H	1	0.000	1.000	18
H ₂ O				20
D	2	0.111	0.889	25
D ₂ O				35
Be	9	0.640	0.360	86
C	12	0.716	0.284	114
O	16	0.779	0.221	150
Na	23	0.840	0.160	218
U	238	0.983	0.017	2148

[†] Average number of collisions to moderate a fast (1 MeV) neutron to a thermal equilibrium energy of 0.025 eV.

- Low Z atoms are more effective moderators than high Z atoms
- Most modern reactors use H, D, or C as moderators



Infinite Systems

- Consider the life cycle of a single fission neutron, and the different paths it can take:
- Born at high energy (fast $> 1\text{MeV}$)
 - Some fast neutrons are absorbed and cause fission
- Interacts with moderator to slow down
 - Some are absorbed by moderator
- Once the neutron reaches thermal energy it is absorbed
 - Only some of the thermal neutrons are absorbed in the fuel.
 - Only some of the thermal neutrons absorbed in the fuel cause fission events

Infinite Systems

- **Four-Factor Formula** for k-infinity

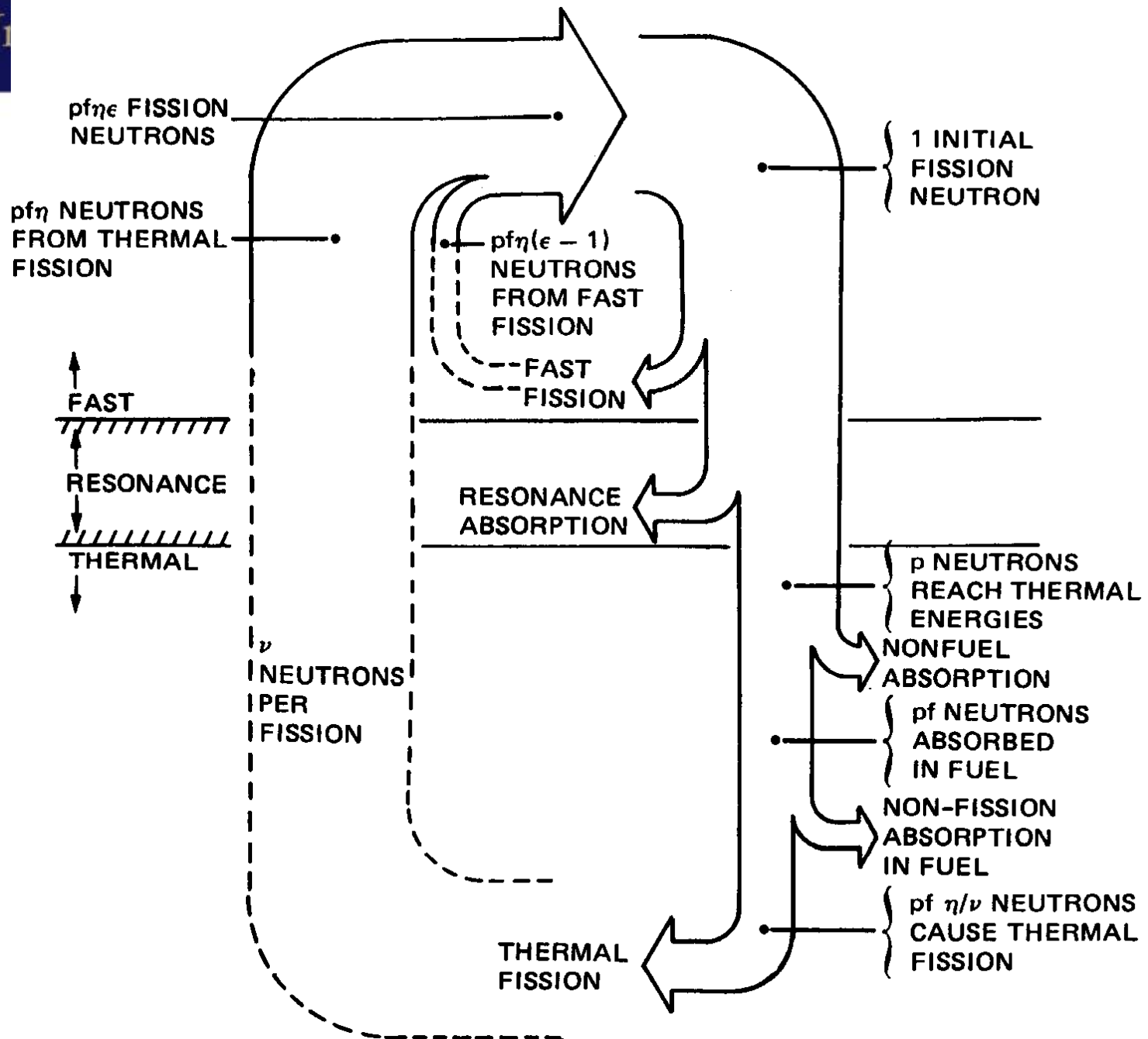
$$k_{\infty} = \varepsilon p \eta f$$

$$\varepsilon = \text{Fast Fission Factor} = \frac{(\nu \Sigma_f)_{total}}{(\nu \Sigma_f)_{th}}$$

$$p = \text{Resonance Escape Probability} = \frac{(\Sigma_a)_{th}}{(\Sigma_a)_{total}}$$

$$f = \text{Thermal Utilization Factor} = \frac{(\Sigma_a)_{th}^{fuel}}{(\Sigma_a)_{th}}$$

$$\eta = \text{"Eta"} = \text{Reproduction Factor} = \frac{(\nu \Sigma_f)_{th}}{(\Sigma_a)_{th}^{fuel}}$$





Infinite Systems

- Four-Factor Formula

$$k_{\infty} = \varepsilon p \eta f$$

$$k_{\infty} = \frac{(\nu \Sigma_f)_{\text{total}}}{(\Sigma_a)_{\text{total}}}$$

$$\varepsilon = \text{Fast Fission Factor} = \frac{(\nu \Sigma_f)_{\text{total}}}{\cancel{(\nu \Sigma_f)_{th}}}$$

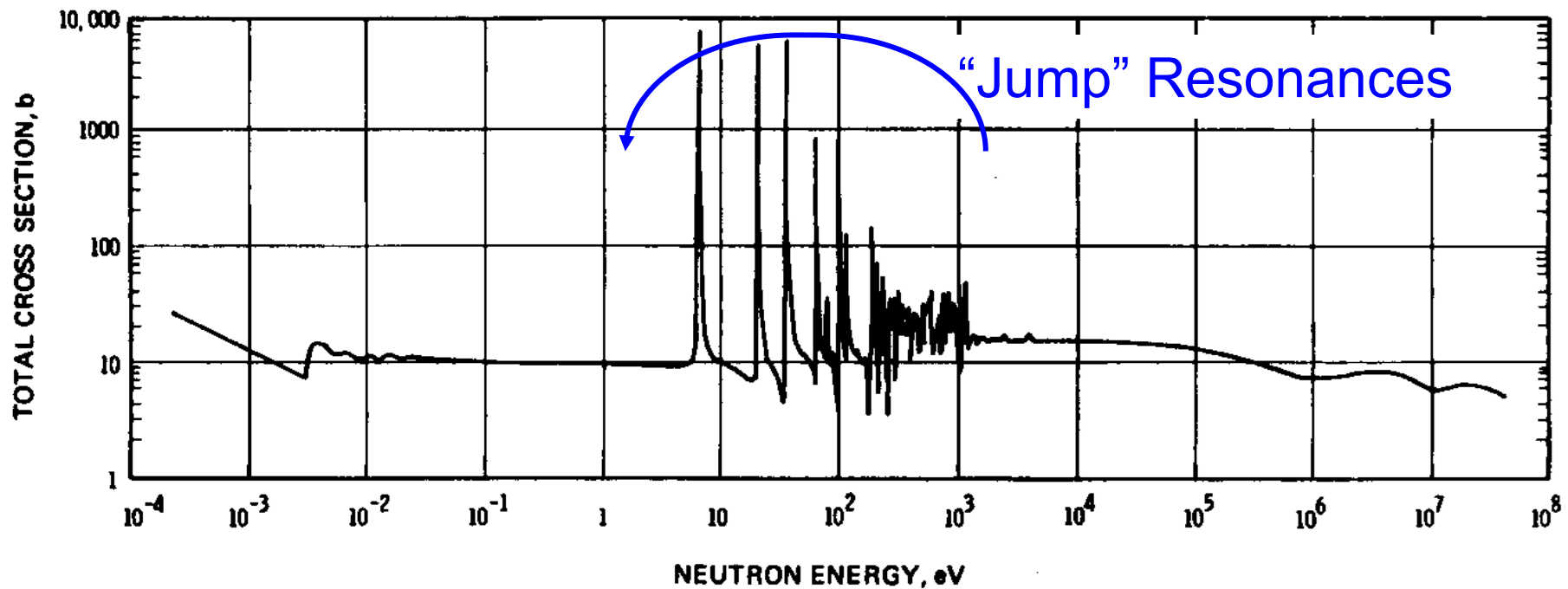
$$p = \text{Resonance Escape Probability} = \frac{\cancel{(\Sigma_a)_{th}}}{(\Sigma_a)_{\text{total}}}$$

$$f = \text{Thermal Utilization Factor} = \frac{\cancel{(\Sigma_a)_{th}^{fuel}}}{\cancel{(\Sigma_a)_{th}}} \quad \left. \vphantom{\frac{\cancel{(\Sigma_a)_{th}^{fuel}}}{\cancel{(\Sigma_a)_{th}}}} \right\} \eta f = \frac{(\nu \Sigma_f)_{th}}{(\Sigma_a)_{th}}$$

$$\eta = \text{"Eta"} = \text{Reproduction Factor} = \frac{\cancel{(\nu \Sigma_f)_{th}}}{\cancel{(\Sigma_a)_{th}^{fuel}}}$$

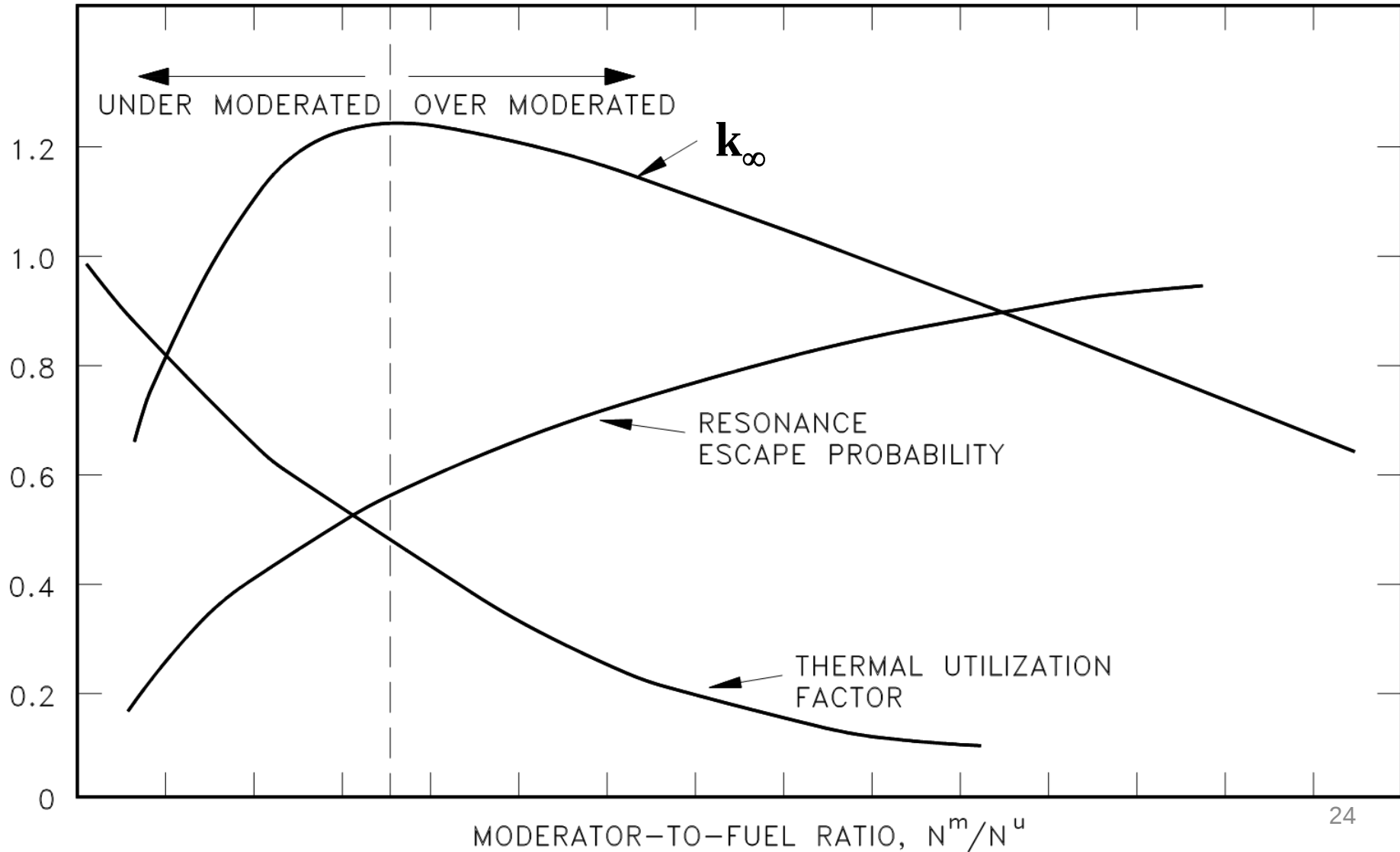
^{238}U Absorption Cross Section

Importance of Resonance Escape





Moderator-to-Fuel Ratio Effect on k_{∞}





Four-Factor Formula

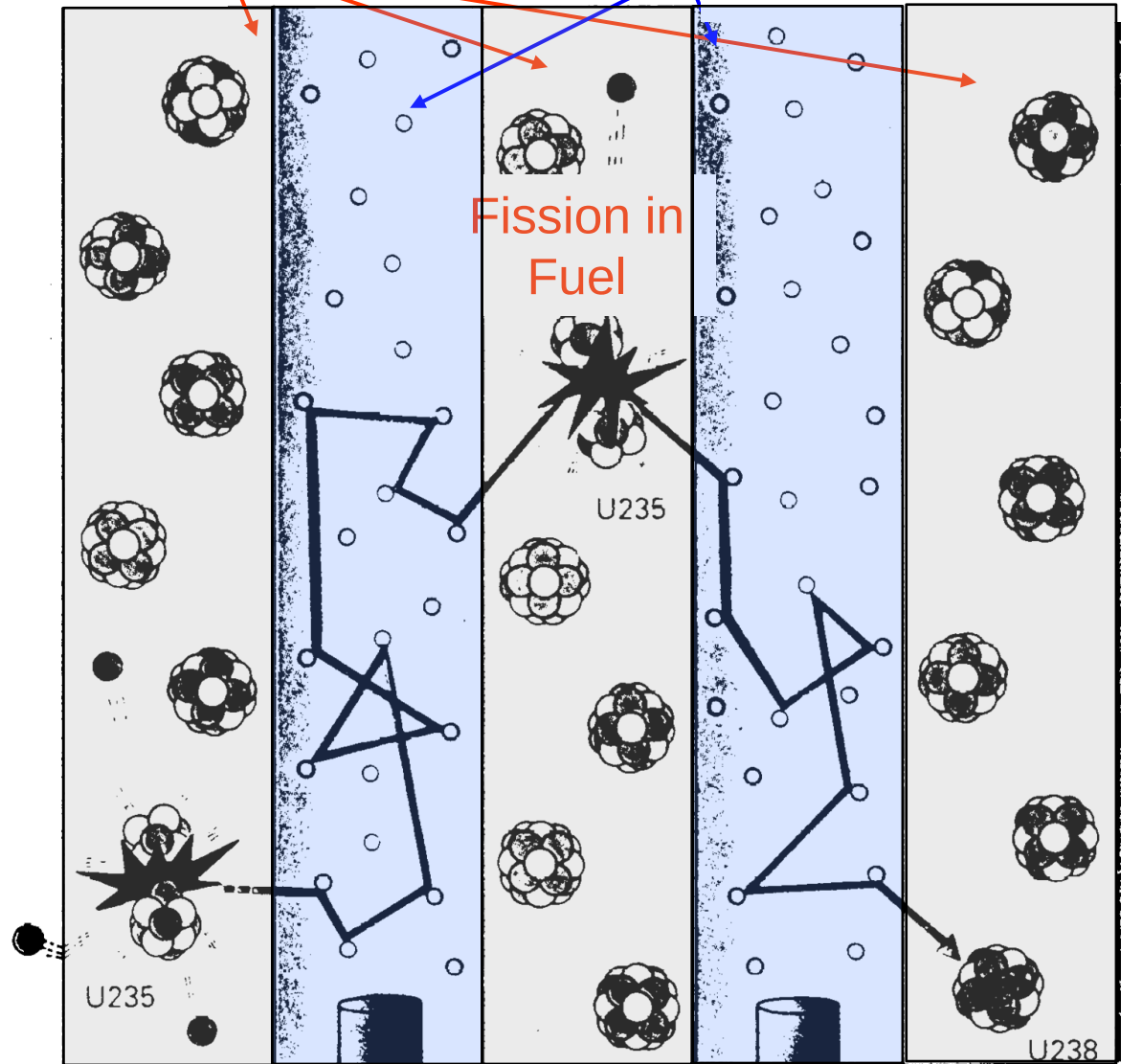
- Fuel “Lumping” / Lattice Arrangement
 - Increase Fast Fission Factor e
 - Increase Resonance Escape Probability p
 - Decrease Thermal Utilization f
 - Pin Diameter and Spacing to optimize $p \times f$
 - Similar to previous k_{∞} vs. M-to-F curve
 - Example: LWR-like lattice

Effect of Heterogeneity of Fuel & Moderator

Fuel Rods

Moderator Channels

Fission in Fuel



Slowing Down in Moderator

Finite System

- Neutron Balance

Production $\leftrightarrow$ Absorption + Leakage

$$\nu \Sigma_f \Phi \leftrightarrow \Sigma_a \Phi \quad + \text{Leakage}$$

$$k_{eff} = k = \frac{\text{Production}}{\text{Absorption} + \text{Leakage}}$$

Note: $k_{\infty} > k_{eff}$ (To accommodate leakage)

Six Factor Formula

- Six-Factor Formula

$$k_{\text{eff}} = k_{\infty} P_{\text{fnl}} P_{\text{tnl}}$$

$$k_{\text{eff}} = \varepsilon p \eta f P_{\text{fnl}} P_{\text{tnl}}$$

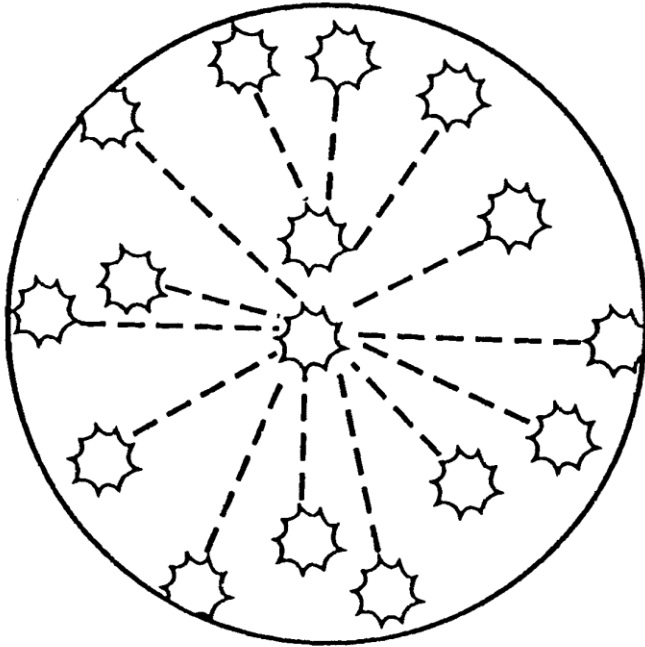
P_{fnl} = Fast Non-Leakage Probability

P_{tnl} = Thermal Non-Leakage Probability

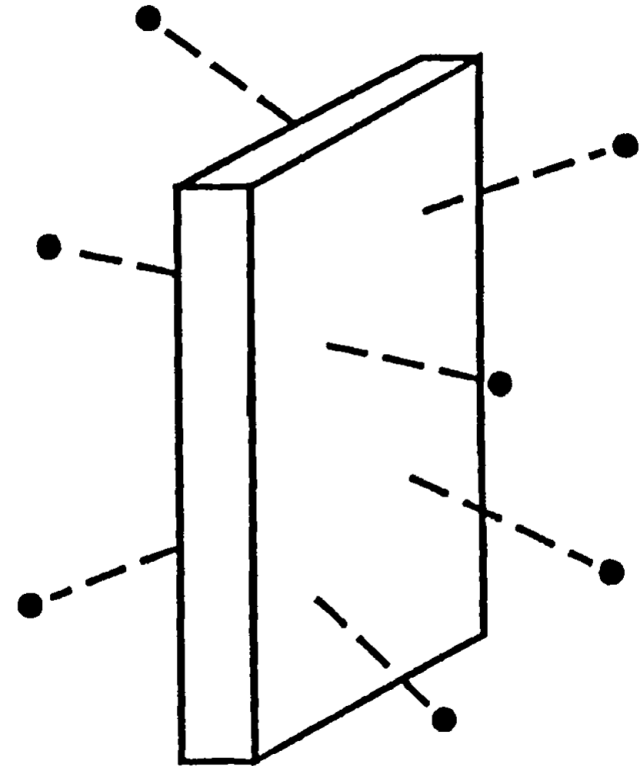
$$k_{\text{eff}} = \varepsilon p \eta f P_{\text{nl}}$$

P_{nl} = Total Non-Leakage Probability

Neutron Leakage Effect



Few Neutrons Leak from
Volume as Sphere

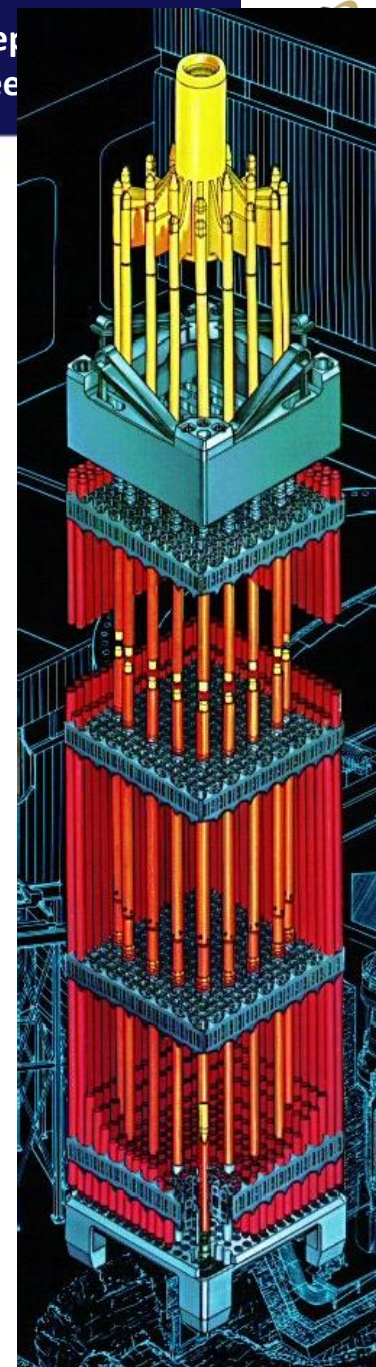
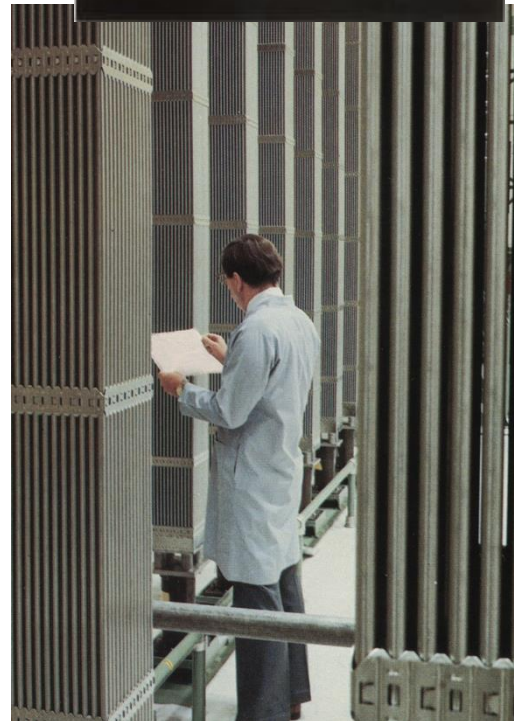
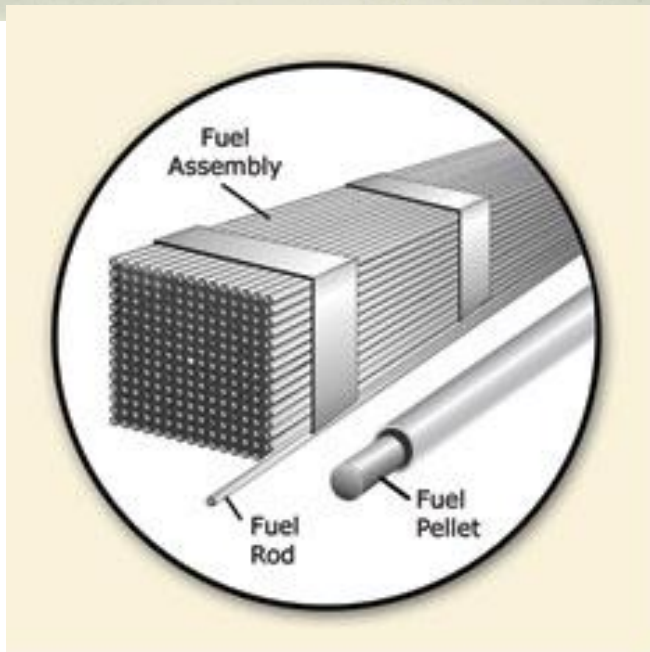
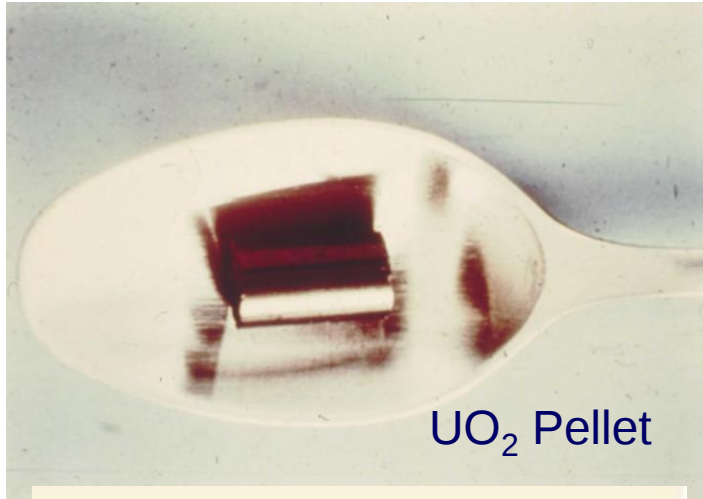


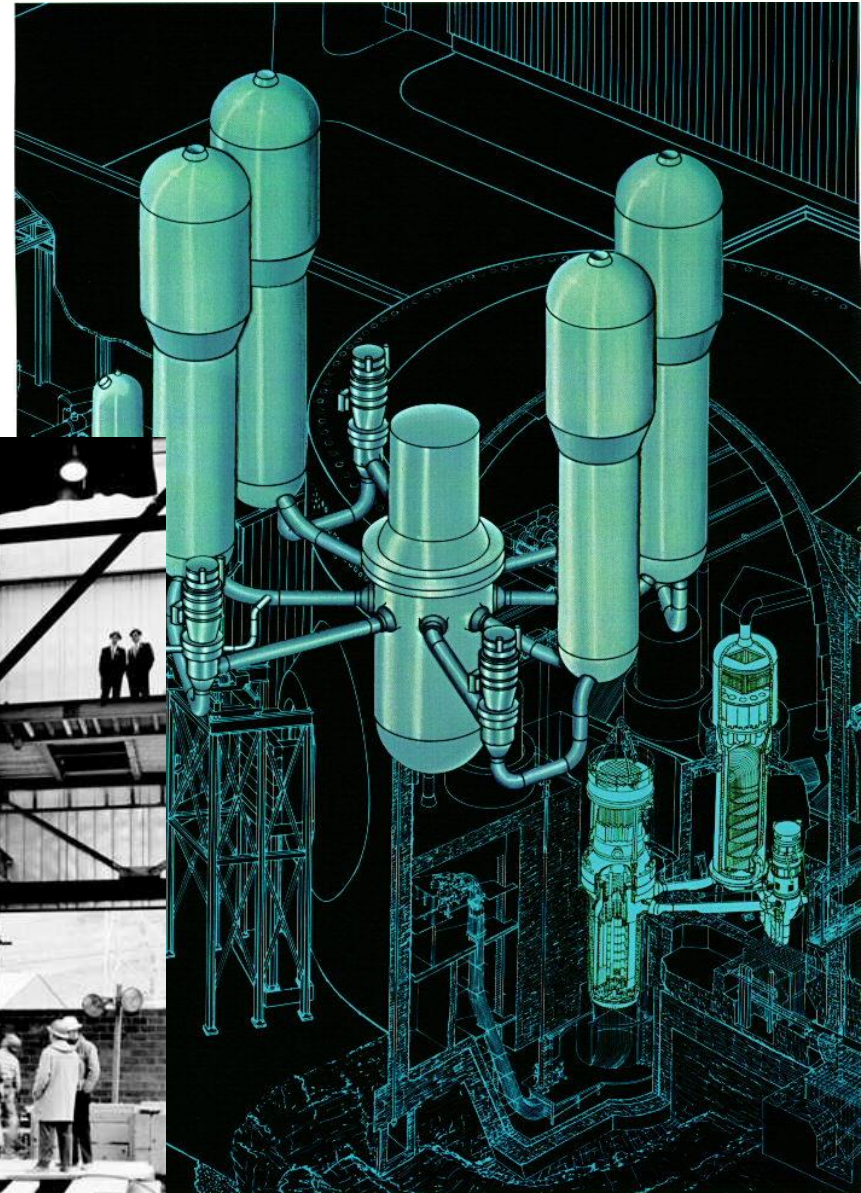
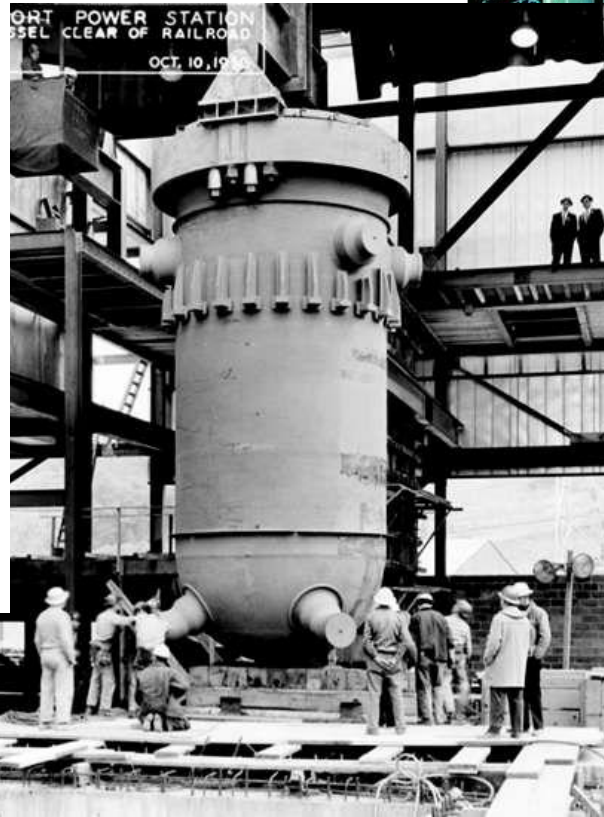
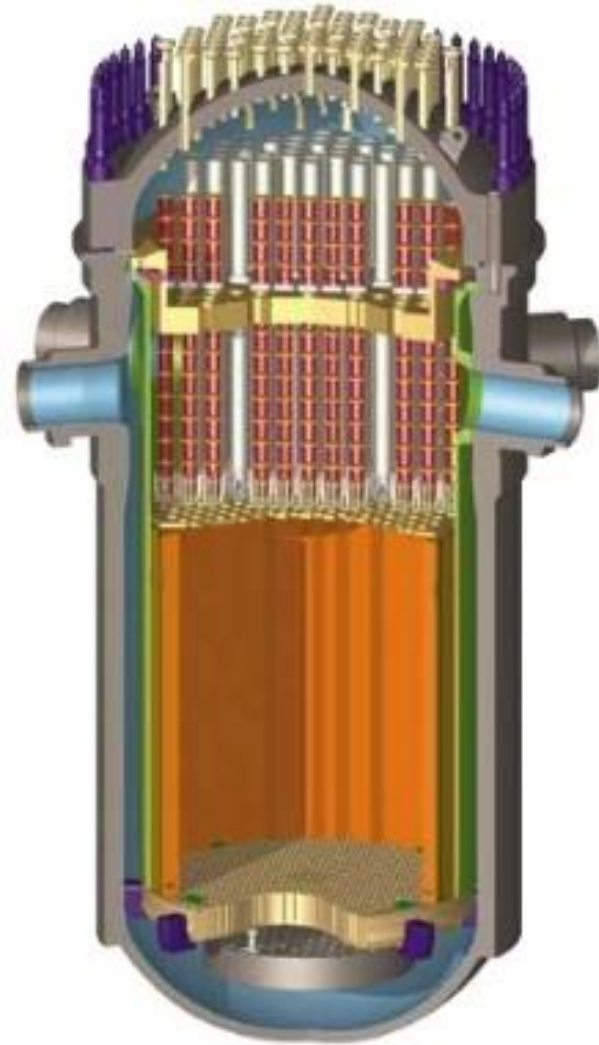
More Neutrons Leak
from Volume as Slab

Leakage depends on shape & size (surface-to-volume ratio)



Reactor Fuel

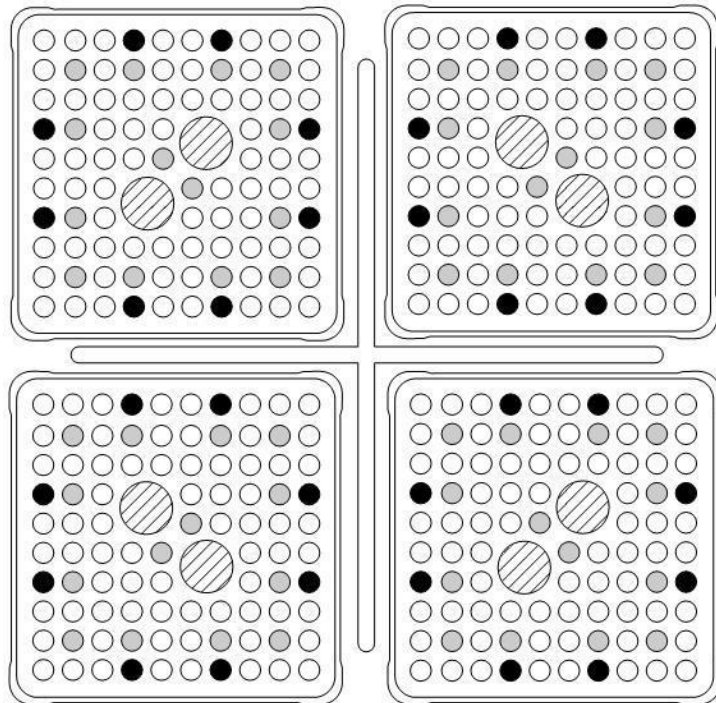




AR STEAM SUPPLY SYSTEM



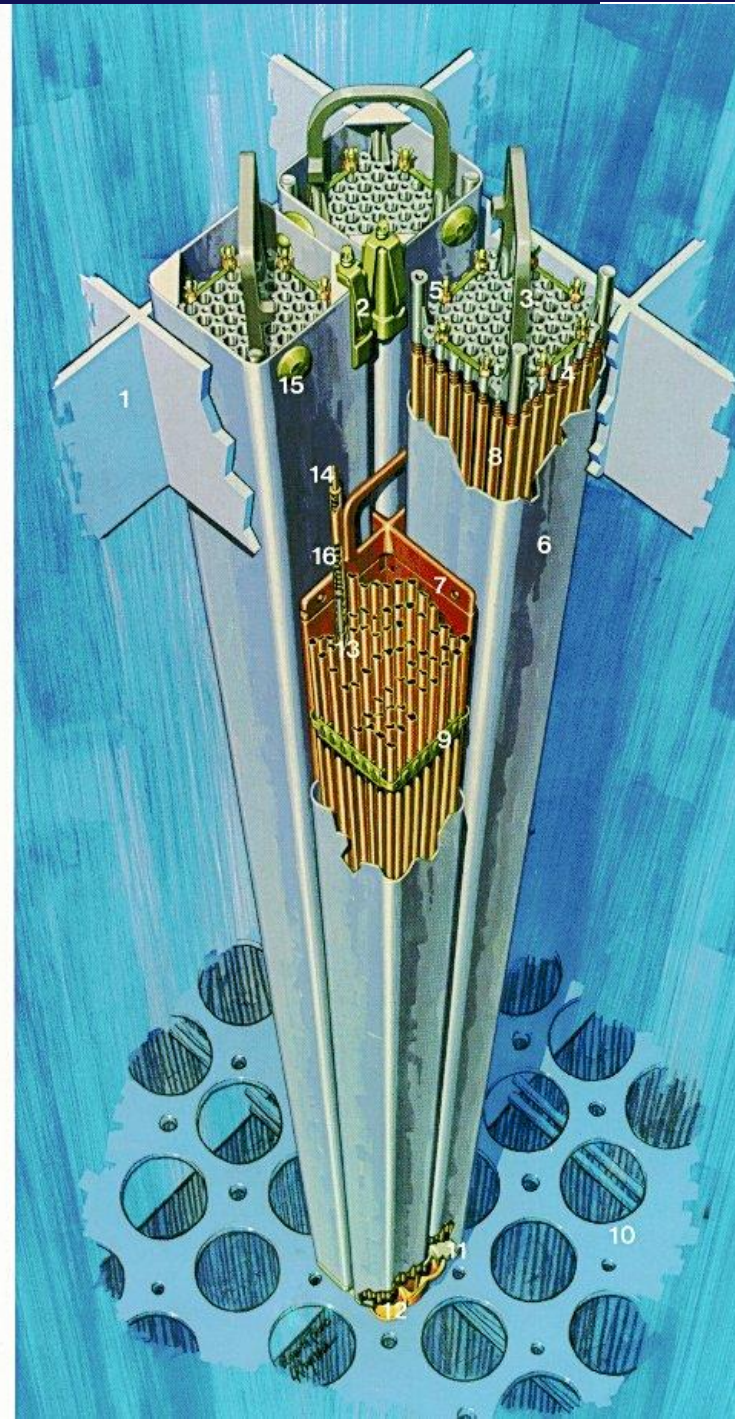
Reactor Fuel



- FUEL ROD
- TIE ROD
- PART LENGTH ROD
- ▨ WATER ROD

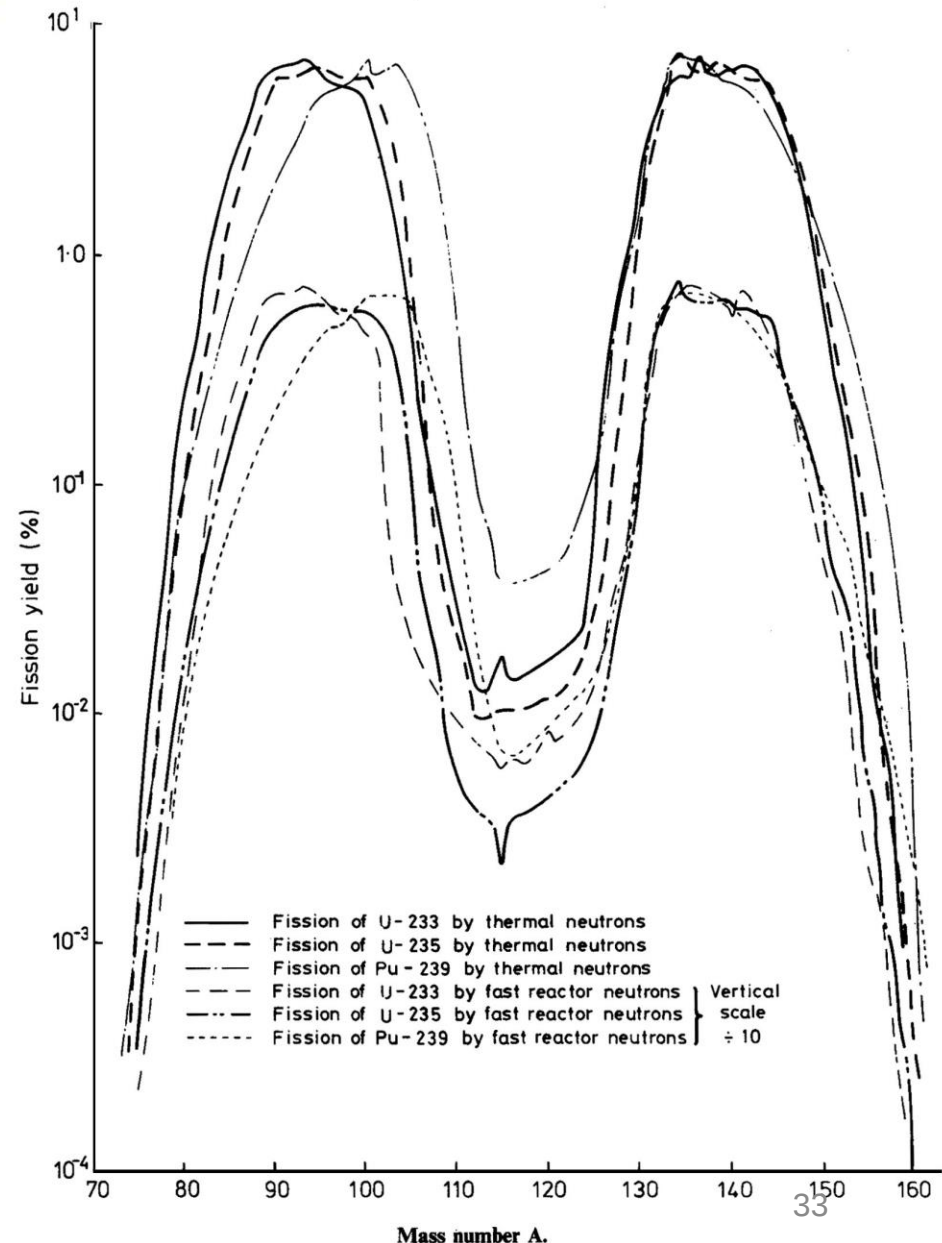
BWR/6 FUEL ASSEMBLIES & CONTROL ROD MODULE

- 1.TOP FUEL GUIDE
- 2.CHANNEL FASTENER
- 3.UPPER TIE PLATE
- 4.EXPANSION SPRING
- 5.LOCKING TAB
- 6.CHANNEL
- 7.CONTROL ROD
- 8.FUEL ROD
- 9.SPACER
- 10.CORE PLATE ASSEMBLY
- 11.LOWER TIE PLATE
- 12.FUEL SUPPORT PIECE
- 13.FUEL PELLETS
- 14.END PLUG
- 15.CHANNEL SPACER
- 16.PLENUM SPRING



Criticality Control

- No reactor can be constantly critical
- Fuel depletion
 - Fission removes a fuel atom and creates two new atoms
 - Transient fission product poisons
 - Xenon and Samarium
 - Fission product poison build up
 - ^{83}Kr , ^{95}Mo , ^{143}Nd , ^{147}Pm
- Temperature (moderator density) changes





Creating Neutron Balance

- States of criticality
 - $k_{eff} = 1$ Critical
 - $k_{eff} > 1$ Supercritical
 - $k_{eff} < 1$ Subcritical
- In order to keep an operating nuclear reactor critical we will need to “adjust” terms in the neutron balance
- Neutron balance controls
 - Production
 - Absorption
 - Leakage



Creating Neutron Balance

- Let us consider how we could adjust these parameters to achieve a target k_{eff} for two different applications
- Nuclear Power Plant
 - Target k_{eff} :
 - $k_{eff} = 1$ for steady-state operation
 - $k_{eff} > 1$ for start-up, $k_{eff} < 1$ for shutdown
- Nuclear Fuel Processing Facility
 - Target k_{eff} :
 - $k_{eff} < 1$ under all possible conditions (including accidents)



Beyond 4 & 6 Factor Formulas

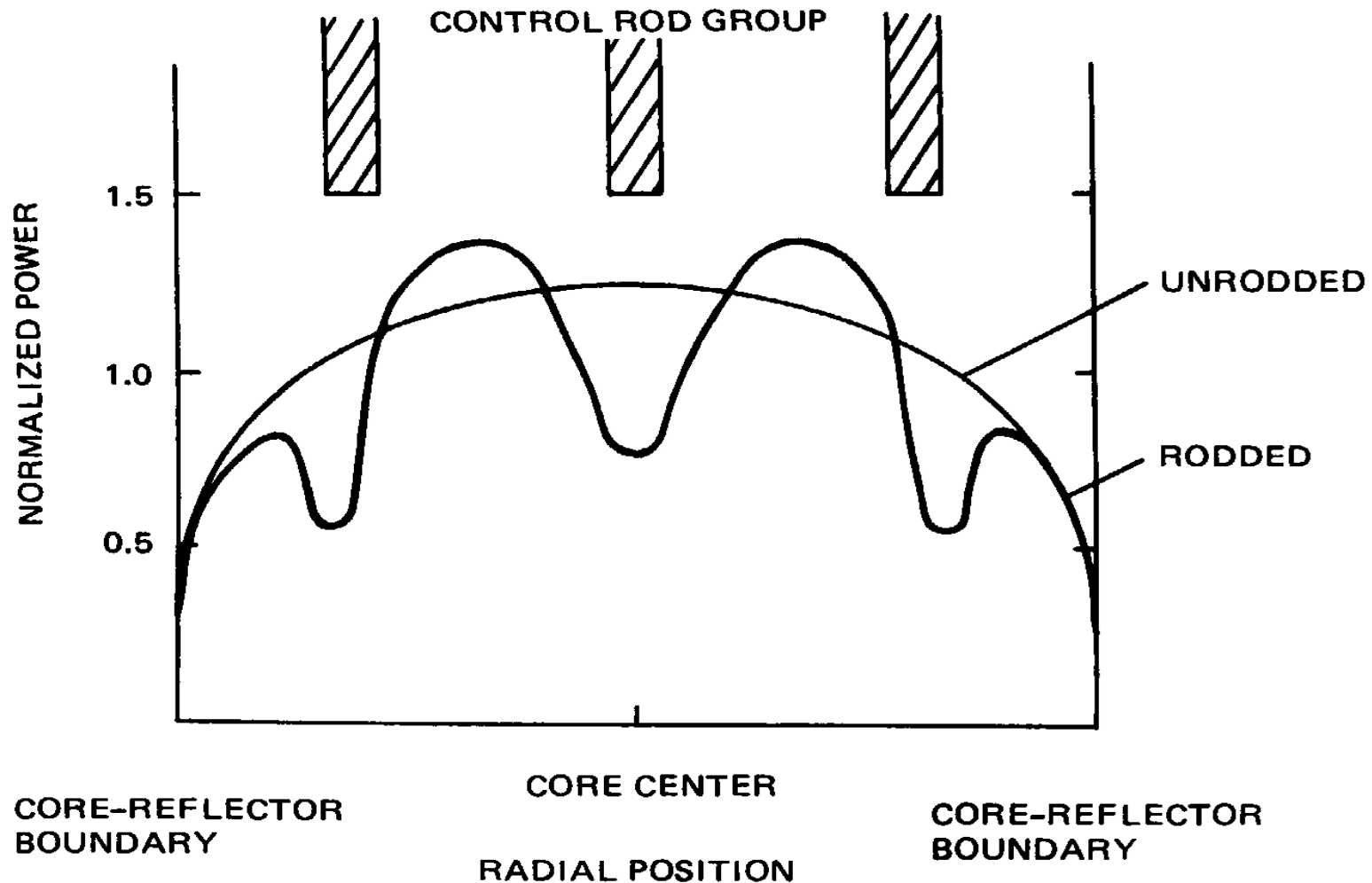
- In a large reactor, different parts of the core may be behaving very differently
 - Outer regions of the core will have a large amount of neutrons escaping from the core, and will be losing a large fraction of neutrons than are born within the region.
 - Neutrons produced in inner regions of the core have little chance of escaping the core. These inner regions will effectively produce more neutrons than are needed locally for fission.
- For the reactor as a whole to be critical, these local regions must balance each other out.
 - Different concentrations of neutron densities and reaction rates throughout the core.
 - Neutrons “flow” from the center of the core towards the edge.



Neutron Density

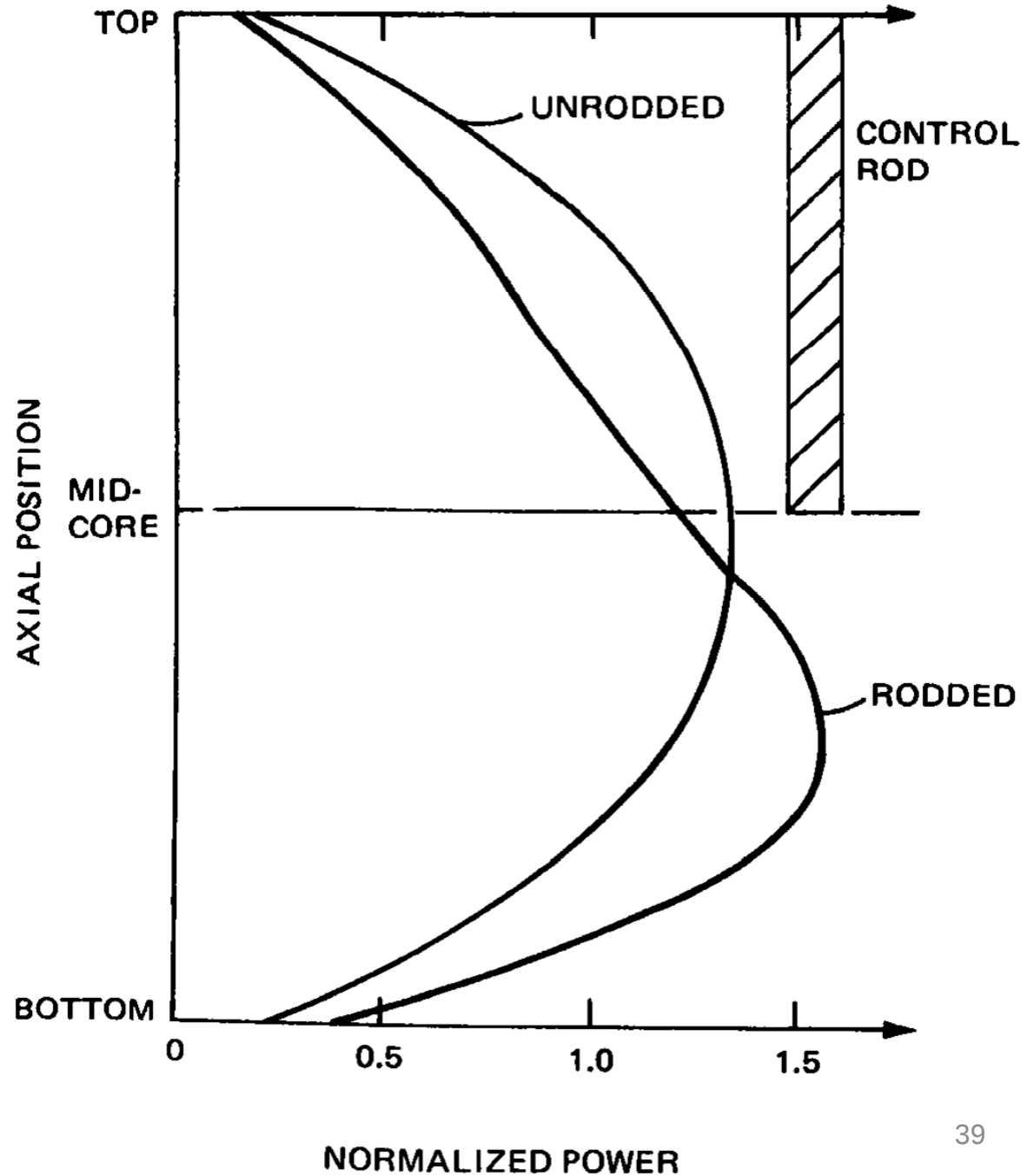
- During steady-state operations there is a natural spatial distribution of neutrons throughout the core.
- This natural distribution depends on the shape of the reactor and the locations of fissile fuel and neutron poisons in the core
 - Peaked in center
 - Low near edge of core
 - Low density near neutron poisons
- In addition to the gross shape of the neutron density, there are local variations that can have a significant effect on the behavior of the core
 - Localized peaking is usually limiting condition in core

Radial Neutron Density





Axial Flux w/ Control Rods



Example of Energy Dependence

C5G7-MOX: Group 1 Scalar Flux

