University of Pittsburgh

ME/ENGR 2100 Fundamentals of Nuclear Engineering

Thermal-Hydraulics Applied to Nuclear Reactors

Module 8 Review

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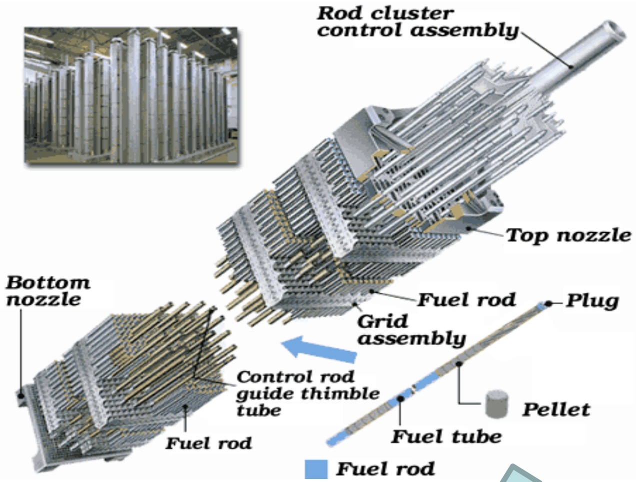
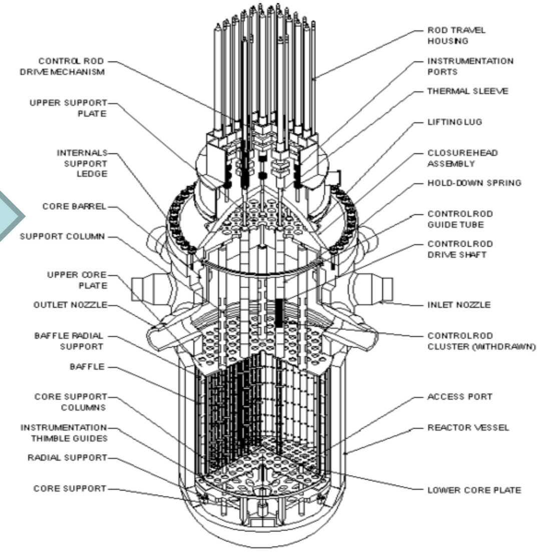
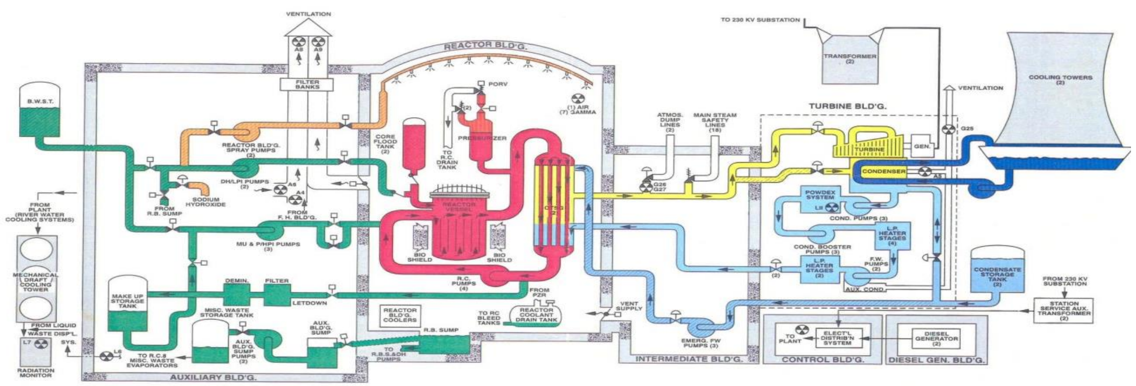
Outline for Thermal-Hydraulics





Neutron Flux Distribution



Nuclear Heat Generation

Nuclear Heat Generation

- Fuel in a nuclear reactor generates heat both during operation and while shutdown
  - During operation → fission heat
  - Shutdown → decay heat
- In order to design a reactor, the rate of heat generation in the fuel at any point in time must be know so that appropriate cooling systems can be designed

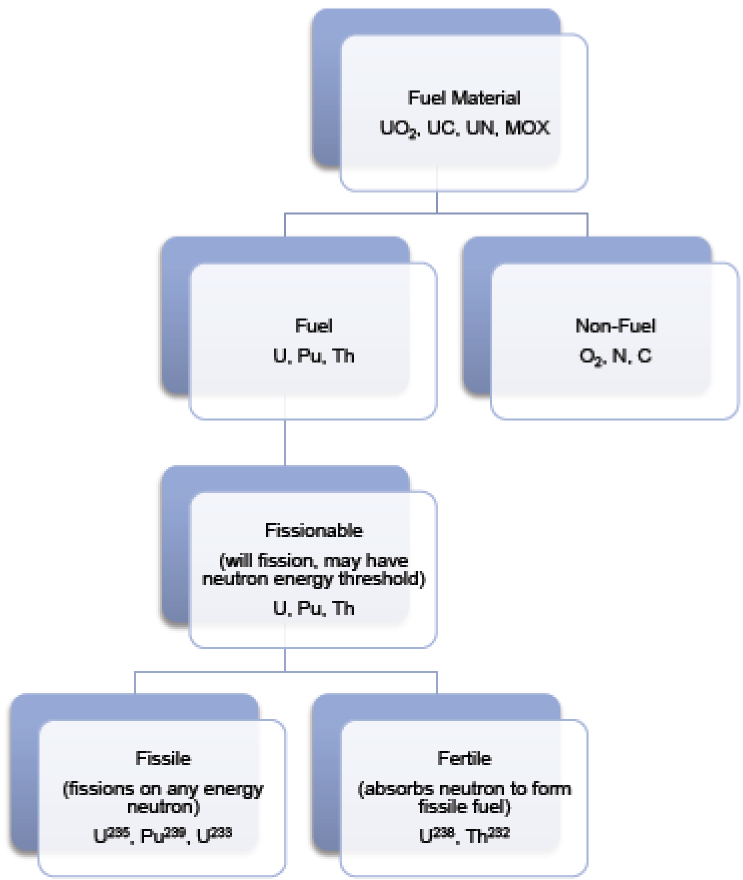
Nuclear Heat Generation

- Definitions
  - Rate of energy generation per fuel rod - $q\left[W, \frac{BTU}{hr}\right]$
  - Volumetric heat rate – $q'''\left[\frac{W}{m^3}, \frac{BTU}{hr-ft^3}\right]$
  - Surface heat flux – $q''\left[\frac{W}{m^2}, \frac{BTU}{hr-ft^2}\right]$
  - Linear heat rate – $q'\left[\frac{W}{m}, \frac{BTU}{hr-ft}\right]$
  - Core power – $Q\left[W, \frac{BTU}{hr}\right]$
  - Core power density – $Q'''\left[\frac{W}{m^3}, \frac{BTU}{hr-ft^3}\right] = \frac{Q}{Vol_{core}}$
  - Core specific power – $Q'''\left[\frac{W}{kg}, \frac{BTU}{hr-lbm}\right] = \frac{Q}{m_{\text{heavy atoms}}}$

Nuclear Heat Generation

- Relationships
  - $$q = \iiint q'''(r)dV = \iint \overrightarrow{q''}(S) \bullet \hat{n} dS = \int q'(z)dz$$
    - Remember, surface heat flux is a vector with a magnitude and direction. $\hat{n}$ is the unit vector normal to the surface the heat is traveling through
- Core
  - $$Q = \sum_{n=1}^{N_{rods}} q_n$$
  - $$Q = N\langle q \rangle = NL_{rod}\langle q' \rangle = N\pi D_{rod} L_{rod}\langle q'' \rangle = N\pi R_{fuel}^2 L_{rod}\langle q''' \rangle$$

Review of Nuclear Fuel Definitions



Fission Heat

	Fission Product	Energy % (MeV)	Description	~ Range	Where
Instantaneous	Fission Fragments	80.5% (161 MeV)	Charged, heavy	0.01 cm	Fuel
	Fast $_0n^1$	2.5% (5 MeV)	Neutral	~10 cm	Moderator
	Fission γ	2.5% (5 MeV)	Neutral	~100 cm	Structure
Delayed	Delay $_0n^1$	0.02% (~0)	Neutral	~10 cm	Moderator
	Fission Frag. β	3% (6 MeV)	Charged	Short	Fuel
	Fission Frag. γ	3.0% (6 MeV)	Neutral	~10 cm	Structure
	Neutrinos	5% (10 MeV)	?	Huge	Who knows
Both	Neutron Capture γ & β	3.5% (8 MeV)			Most in fuel

Tally

Fuel
~ 174 MeV

Moderator
~ 5 MeV

Structure
~ 11 MeV

Lost
~ 10 MeV

Fission Heat

- The volumetric fission rate $\left(R_{fission}\right)$ is the product of the neutron flux (ϕ) and the effective $\left(\overline{\Sigma}_f\right)$ macroscopic fission cross section

$$q'''_{fuel} = R_{fission} Q_{fission} = \overline{\Sigma}_f \phi Q_{fission}$$

  - Reminder: The macroscopic cross section is the product of the number density $\left(N_{ff}\right)$ and microscopic cross section $\left(\overline{\sigma}_f\right)$

$$q''' = \overline{\Sigma}_f \phi Q_{fission} = N_{ff} \overline{\sigma}_f \phi Q_{fission}$$

Fission Heat

- Fissionable fuel number density $\left(N_{ff}\right)$
  - Function of:
    - Fissionable fuel used(U, Pu, or Th)
    - Enrichment
    - Fuel material (UO_2, U+ZrH, etc.) density
      - Does not include cladding or other structural materials

$$N_{ff} = \frac{A_v}{M_{ff}} \rho_{ff} i$$

- where: A_v is Avogadro's number, M_{ff} is the molecular mass of the fissionable fuel, ρ_{ff} is the density of fissionable fuel, and i is the number of fuel atoms per molecule of fuel

Fission Heat

- Fissionable fuel number density $\left(N_{ff}\right)$
  - The density of the fissionable fuel $\left(\rho_{ff}\right)$ is typically unknown. Can be calculated from either:
    - The fuel density and enrichment, or

$$\rho_{ff} = r\rho_f$$

    - The fuel material density, fuel mass fraction, and enrichment

$$\rho_{ff} = rf\rho_{fm}$$

    - where: r is the enrichment (mass ratio) and f is the mass fraction of fuel in the fuel material

Fission Heat

- Fuel mass fraction (f) cont.
  - When the molecular masses of the fissionable fuel and nonfissionable fuel are approximately equal (ex: U^{235} and U^{238}), simplifies to:

$$f = \frac{rM_{ff} + (1-r)M_{nf}}{rM_{ff} + (1-r)M_{nf} + M_{other}}$$

Example

- Calculate the fissionable fuel number density for a reactor fueled with U/PuO_2. The fuel is enriched to 27%. Assume all of the U is U^{238} and the density of the fuel material is 10.5 g/cm^3.

  – Ans: $N_{ff} = 6.296 \times 10^{21}$ atoms/cm^3

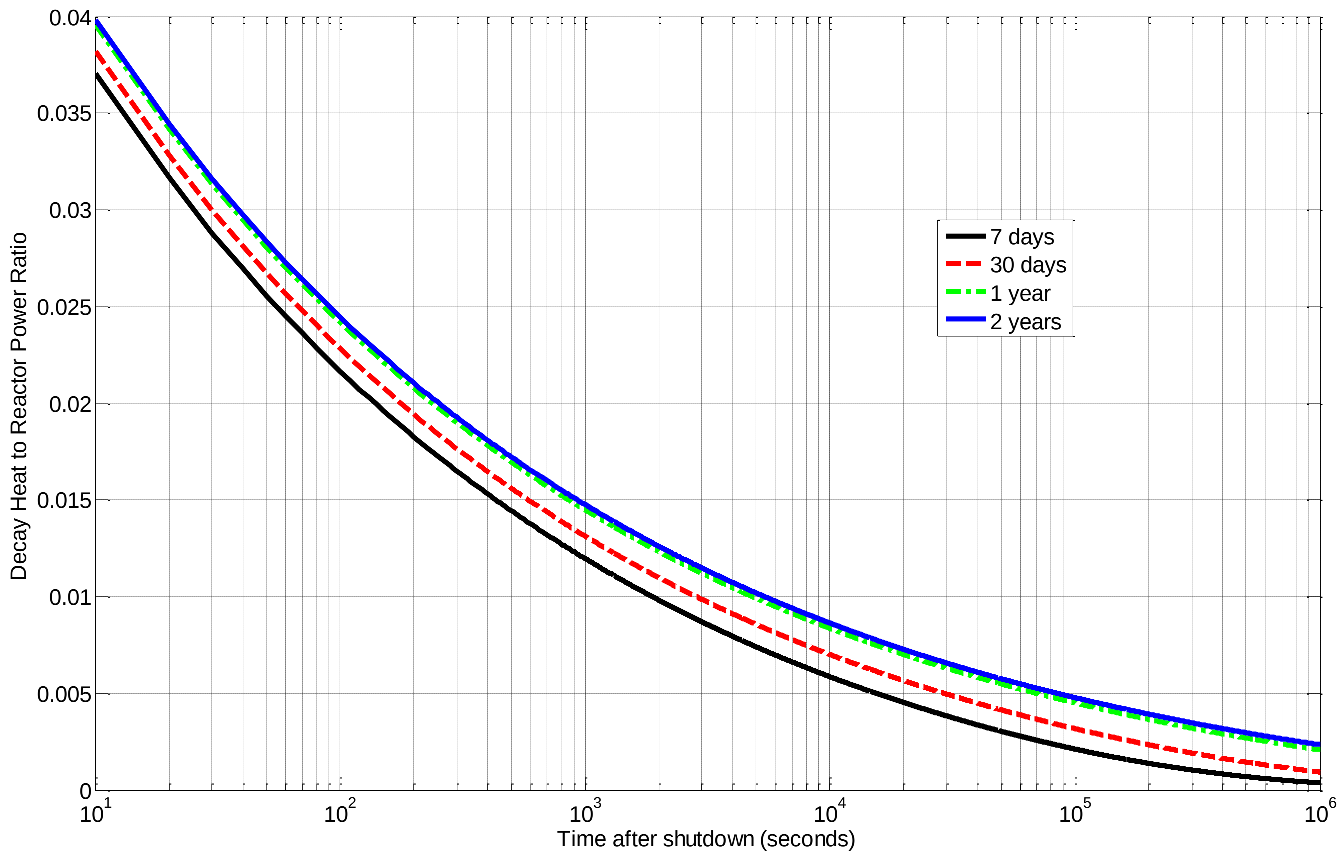
Decay Heat

- Following shutdown of the reactor, power does not immediately drop to zero
  - Falls off rapidly following shutdown
    - Rate determined by the half-life of the longest lived delayed neutron (neutrons emitted by neutron decay of fission products) group
  - Power is still produced by the decay of fission products
    - Beta and gamma decays are major contributors

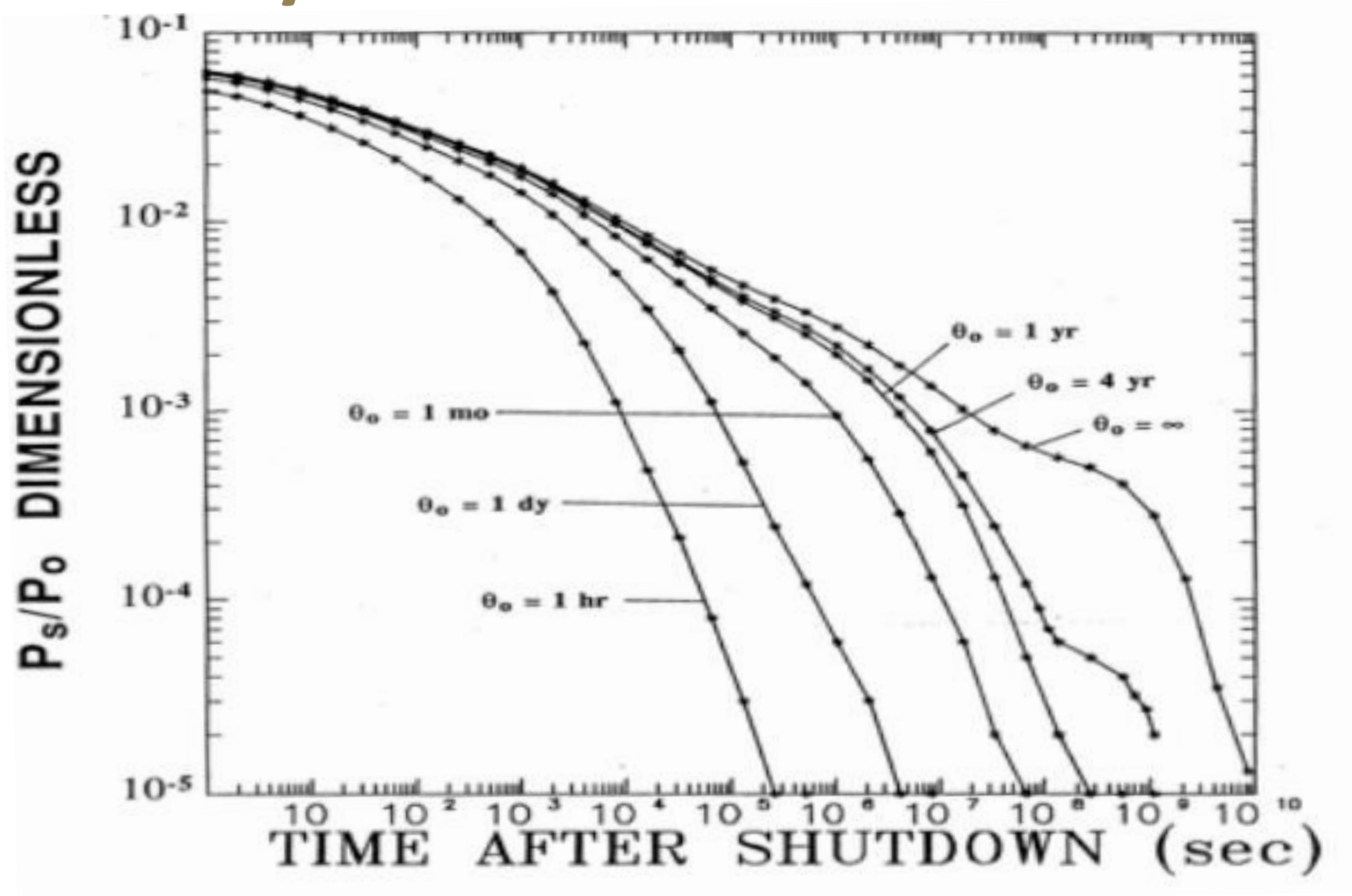
Decay Heat

- Amount of decay heat is dependent upon the operating history of the reactor
  - Longer operation builds up more fission products to decay and produces more decay heat
  - Larger power levels yield the same effect
- Because of the large amount of decay heat produced following shutdown, cooling is still essential to prevent fuel damage
  - Residual Heat Removal (RHR) systems incorporated to provide shutdown cooling
- Has decay heat caused major problems in the nuclear industry?

Decay Heat


0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Decay Heat to Reactor Power Ratio
10^1
10^2
10^3
10^4
10^5
10^6
Time after shutdown (seconds)
7 days
30 days
1 year
2 years

ANS Decay Heat Standard



Other Important Parameters

(1) Reactor thermal power [MW_t]: The total heat produced in the reactor core.

(2) Plant electrical output [MW_e]: Net electrical power generated by the plant.

(3) Net plant efficiency [%]: $$\frac{\text{Plant Electrical Output}}{\text{Reactor Thermal Power}}$$

(4) Plant capacity factor [%]: $$\frac{\text{Total Energy Generated Over Time Period}}{\text{Plant Rating x Time}}$$

(5) Plant load factor [%]: $$\frac{\text{Average Plant Electrical Power Level}}{\text{Peak Power Level}}$$

Other Important Parameters

(6) Plant availability factor [%]: $$\frac{\text{Integrated Electrical Energy Output Capacity}}{\text{Total Rated Energy Capacity for Period}}$$

(7) Core power density [kW/liter]: $$\frac{\text{Reactor Thermal Power}}{\text{Total Core Volume}}$$

(8) Linear power [kW/ft]: $$\frac{\text{Thermal Heat Generated}}{\text{Unit Length of Fuel}}$$

(9) Fuel loading [kg]: Total mass of fuel (i.e., fissionable material)

(10) Specific power [kW/kg]: $$\frac{\text{Reactor Thermal Power}}{\text{Fuel Loading}}$$

Other Important Parameters

(11) Fuel burnup [MW-days/metric ton uranium = MWD/TU]:

$$\frac{\text{Energy Generated in Fuel During Core Residence}}{\text{Fuel Loading}}$$

(12) Fuel residence time:

$$\frac{\text{Fuel Burnup}}{\text{(Specific Power) * (Capacity Factor)}}$$

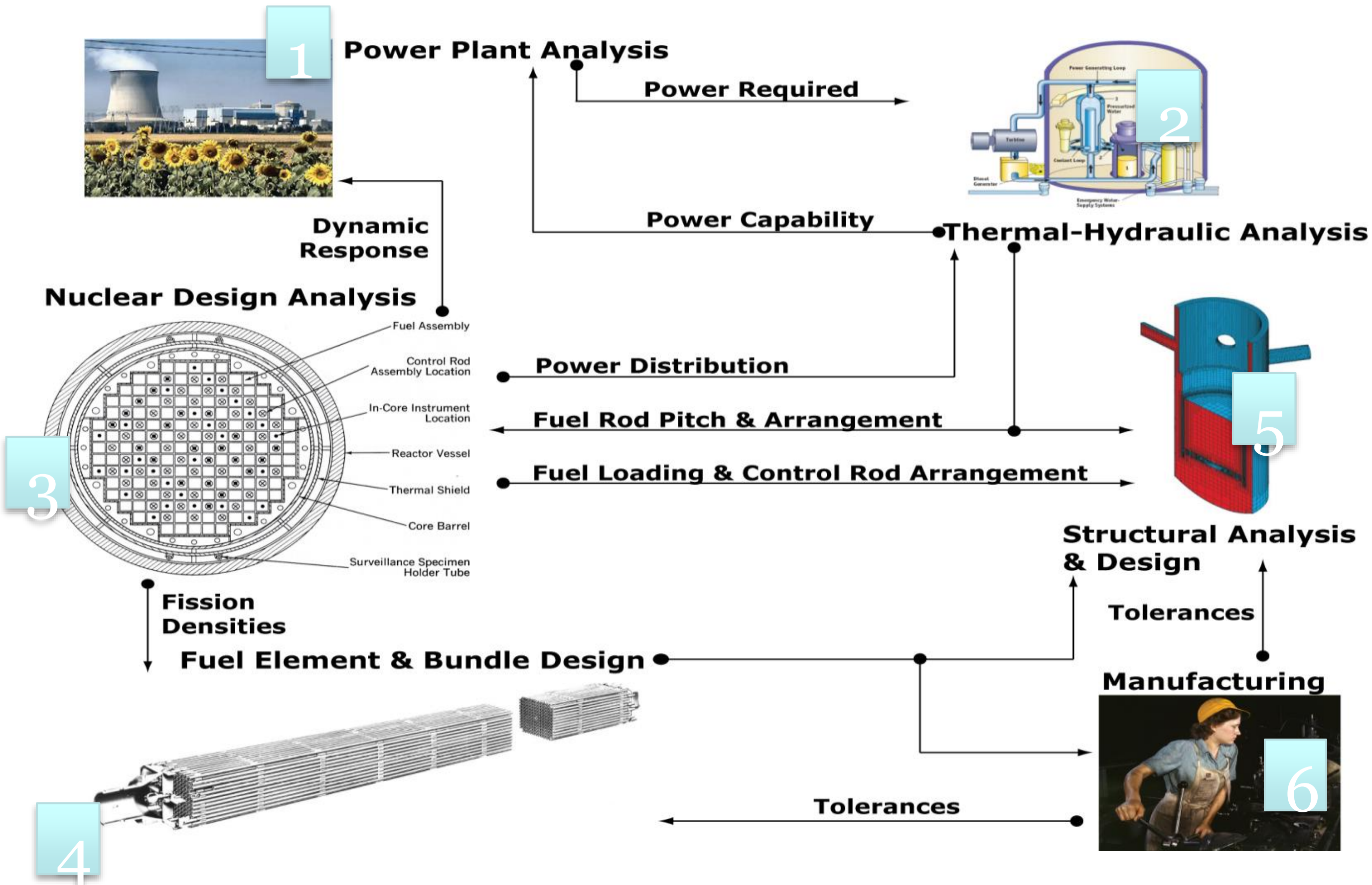
(13) Heat Rate:

$$\frac{\text{Amount of Heat Supplied}}{\text{Needed to Generate 1 kWh of Electricity}}$$

Nuclear Reactor Design

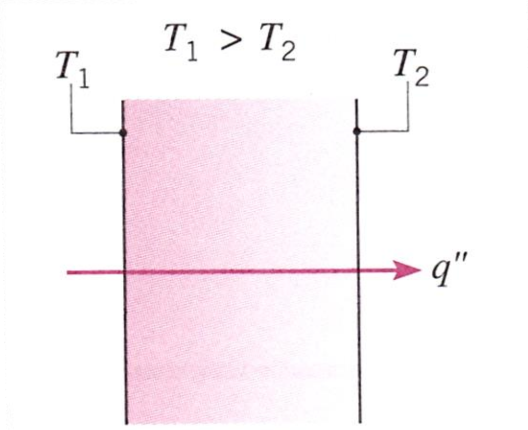
Nuclear Reactor Design

An Iterative Process

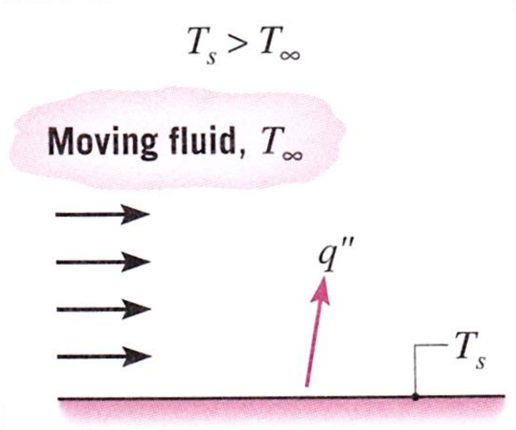


Heat Transfer Mechanisms

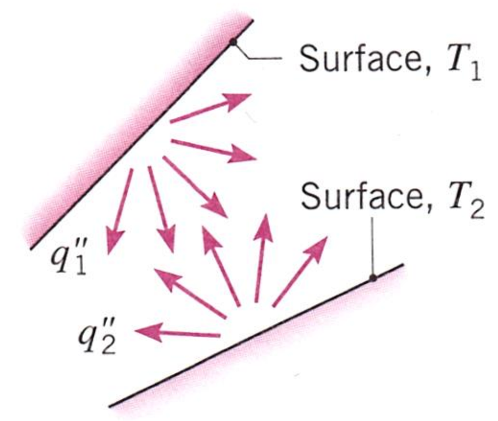
Heat Transfer Modes



Conduction



Convection



Thermal Radiation

Requires:
- Temperature Difference
- Medium to transfer heat

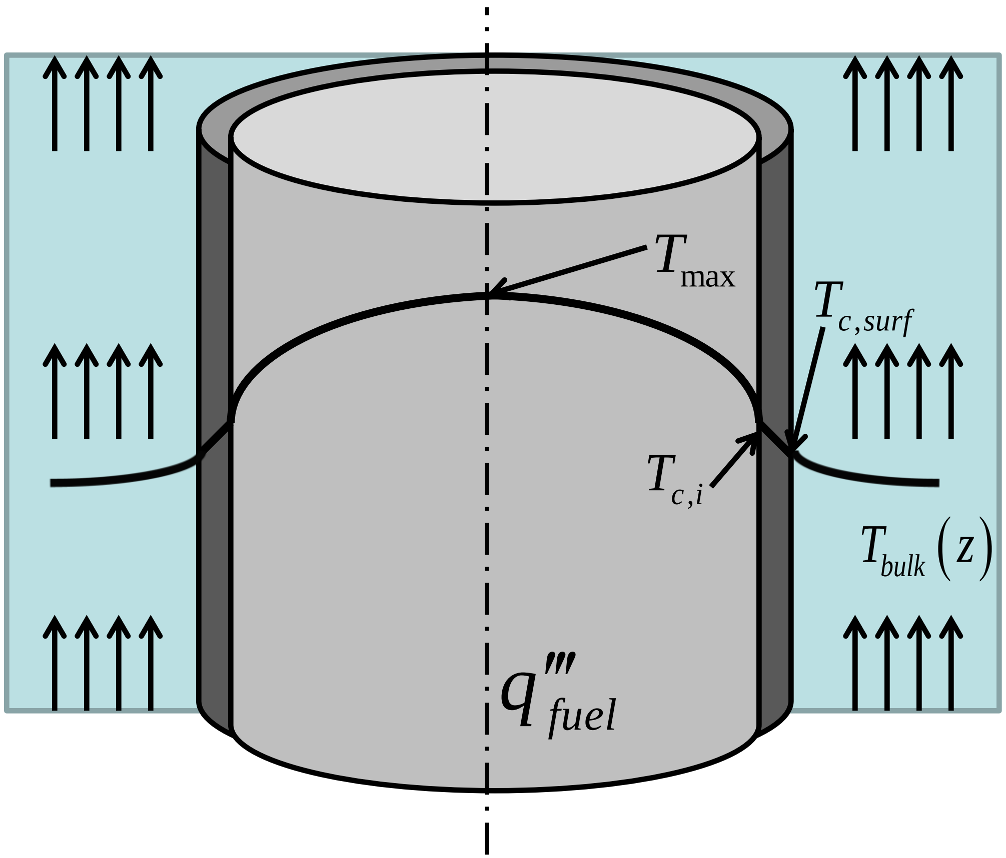
Requires:
- Temperature Difference
- Flowing or moving medium

Requires:
- Temperature Difference

Where does each mode apply to the analysis of a nuclear reactor?

Graphics from: "Incropera, F.P. and Dewitt, D.P., Fundamentals of Heat and Mass Transfer, 5th ed. , John Wiley & Sons, 2002

Nuclear Reactor Thermal Analysis



Idealized Fuel Rod

- Need to obtain temperature distribution within fuel element
  - Heat generated and transferred in fuel pellet
  - Heat transferred through cladding
  - Heat transferred to coolant
- Let's start with the fuel pellet

Derivation of the Conduction Equation

Heat Conduction Equation

$$\frac{d}{dt}\iiint_V (\rho u)\,dV + \iint_S \rho u\left(\vec{v}-\vec{v_s}\right)\bullet\vec{n}dS = \oiint_S\left[-\vec{q''}+\left(\bar{\bar{\tau}}-p\bar{\bar{I}}\right)\bullet\vec{v}\right]\bullet\vec{n}dS + \iiint_V \rho\vec{g}\bullet\vec{v}dV + \iiint_V q'''dV$$

Time rate of change of energy in the volume

Rate of energy loss by convection

Surface heat addition(s)

Viscous heating

Pressure work

Work due to body force

Volumetric heat generation

- Assumptions
  - Solid $\vec{v}=0$
  - No viscous heating and constant pressure $\oiint_S\left[\left(\bar{\bar{\tau}}-p\bar{\bar{I}}\right)\bullet\vec{v}\right]\bullet\vec{n}dS=0$
  - Control volume constant $\vec{v_s}=0$

$$\frac{d}{dt}\iiint_V (\rho u)\,dV = \oiint_S -\vec{q''}\bullet\vec{n}dS + \iiint_V q'''dV$$

Heat Conduction Equation

$$\frac{d}{dt}\iiint_V (\rho u)\, dV = \iint_S -\vec{q''} \bullet \vec{n} dS + \iiint_V q''' dV$$

- From Gauss's Divergence Theorem

$$\iint_S -\vec{q''} \bullet \vec{n} dS = \iiint_V -\vec{\nabla} \bullet \vec{q''} dV$$

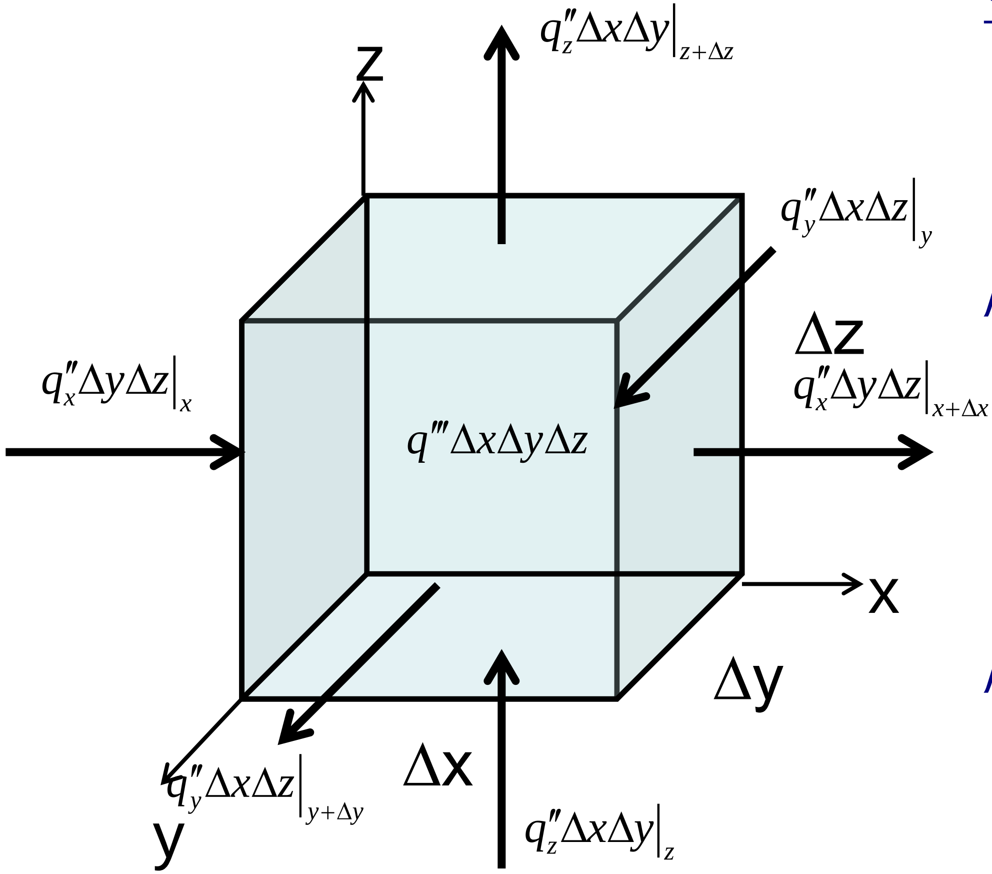
  - Means the volume integral of the divergence of the heat flux vector is equal to the total flux of the vector at the surface

$$\frac{d}{dt}\iiint_V (\rho u)\, dV = \iiint_V -\vec{\nabla} \bullet \vec{q''} dV + \iiint_V q''' dV$$

$$\frac{dE}{dt} = \frac{dmc_p T}{dt} = \frac{d\rho c_p T\,(Vol)}{dt} = -\vec{\nabla} \bullet \left(-k\vec{\nabla}T\right) Vol + q''' Vol$$

$$\rho c_p \frac{dT}{dt} = k\nabla^2 T + q'''$$

Conduction Equation



$$\frac{\partial E}{\partial t}=\frac{\partial mc_pT}{\partial t}=\frac{\partial \rho c_pT}{\partial t}(\Delta x\Delta y\Delta z)$$

$$\rho c_p\frac{\partial T}{\partial t}Vol=\begin{bmatrix} q_x''\Delta y\Delta z\big|_x-q_x''\Delta y\Delta z\big|_{x+\Delta x}+ \\ q_y''\Delta x\Delta z\big|_y-q_y''\Delta x\Delta z\big|_{y+\Delta y}+ \\ q_z''\Delta x\Delta y\big|_z-q_z''\Delta x\Delta y\big|_{z+\Delta z}+ \\ q'''\Delta x\Delta y\Delta z \end{bmatrix}$$

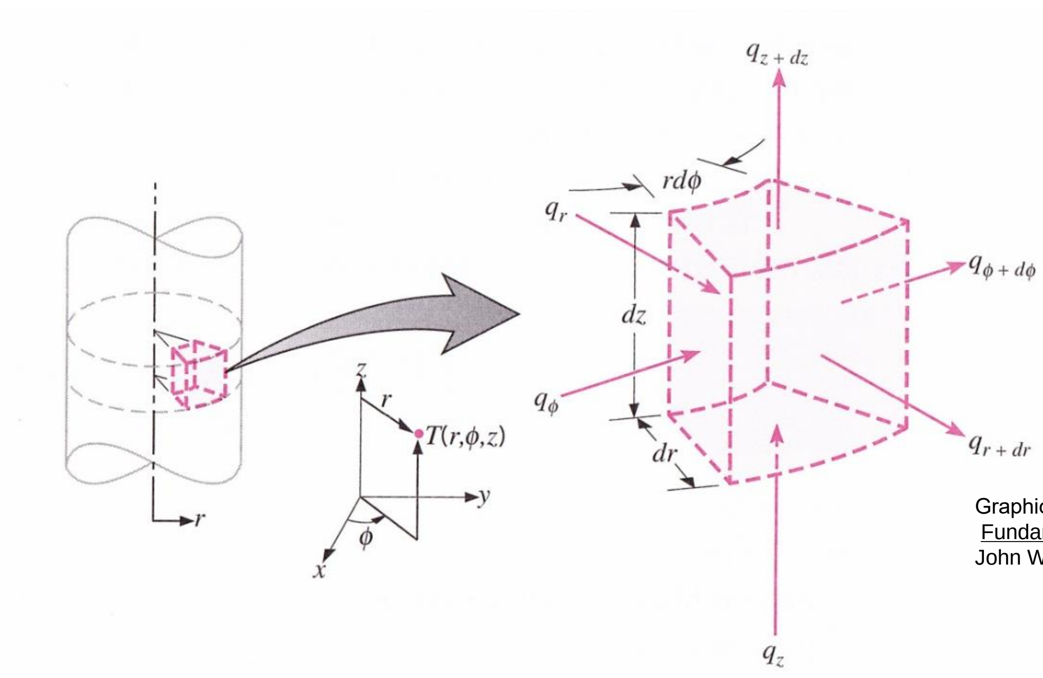
$$\rho c_p\frac{\partial T}{\partial t}=\begin{bmatrix} \dfrac{q_x''\big|_x-q_x''\big|_{x+\Delta x}}{\Delta x}+\dfrac{q_y''\big|_y-q_y''\big|_{y+\Delta y}}{\Delta y} \\ +\dfrac{q_z''\big|_z-q_z''\big|_{z+\Delta z}}{\Delta z}+q''' \end{bmatrix}$$

$$\rho c_p\frac{\partial T}{\partial t}=-\frac{\partial}{\partial x}\left(q_x''\right)-\frac{\partial}{\partial y}\left(q_y''\right)-\frac{\partial}{\partial z}\left(q_z''\right)+q'''$$

Cylindrical Coordinates

$$k\vec{\nabla}T = k\frac{\partial T}{\partial r}\hat{i} + \frac{k}{r}\frac{\partial T}{\partial \phi}j + k\frac{\partial T}{\partial z}k$$

$$\nabla \cdot \left(\nabla kT\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$



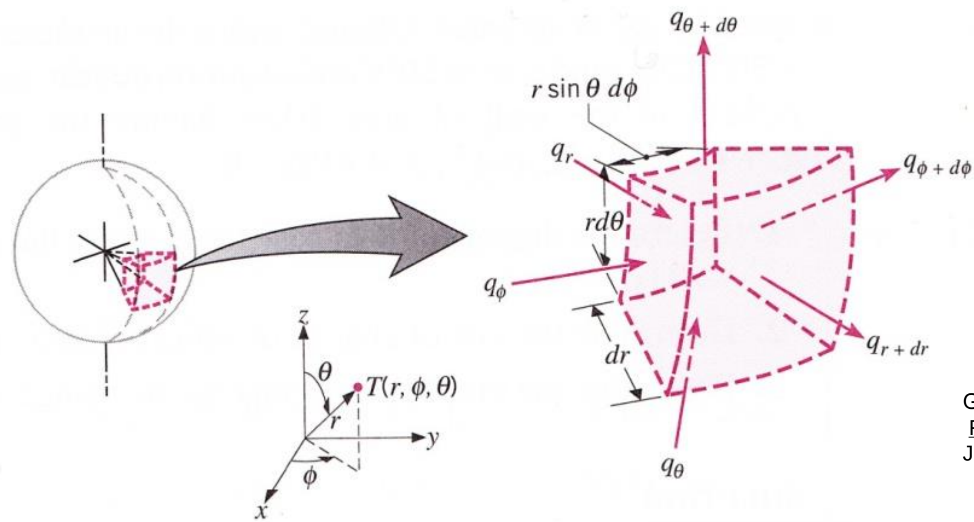
Graphics from: "Incropera, F.P. and Dewitt, D.P., Fundamentals of Heat and Mass Transfer, 5th ed. , John Wiley & Sons, 2002

Spherical Coordinates

- Can be derived in a similar manner.

$$k\vec{\nabla}T = k\frac{\partial T}{\partial r}\hat{i} + \frac{k}{r}\frac{\partial T}{\partial \theta}\,j + \frac{k}{r\sin\theta}\frac{\partial T}{\partial \phi}\,k$$

$$\nabla\cdot\left(\nabla kT\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)$$



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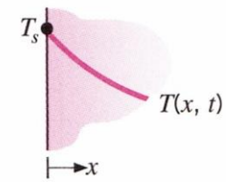
Initial & Boundary Conditions

1. Constant surface temperature

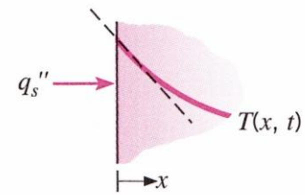
$$T(0, t) = T_s \qquad (2.24)$$



2. Constant surface heat flux

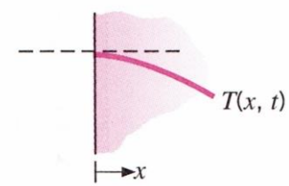
(a) Finite heat flux

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.25)$$



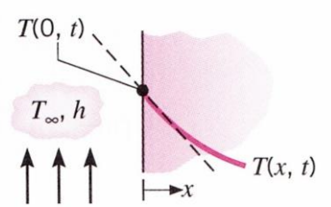
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.26)$$



3. Convection surface condition

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.27)$$



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Interface Constraints

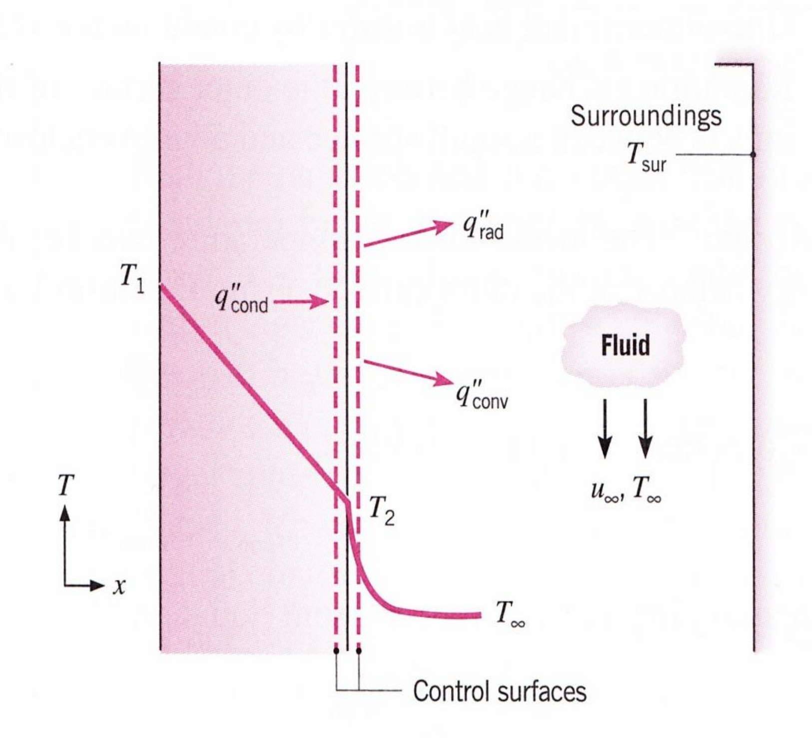
- At interfaces between different materials (i.e., fuel & cladding, pipe & insulation, etc.), must have:
  - Continuous temperature

$$T_{left} = T_{right}$$

  - Continuous Heat Flux

$$q''_{left} = q''_{right}$$

$$-k_{left} \left.\frac{dT_{left}}{dr}\right|_{\text{interface}} = -k_{right} \left.\frac{dT_{right}}{dr}\right|_{\text{interface}}$$



Example 1

- A copper rod is electrically heated such that its volumetric heat generation rate is 670 kW/m^3. The rod is 20cm in diameter and the surface temperature of the rod is 140° C. The conductivity of the cooper is constant at 390 W/m-K. Determine the steady-state temperature distribution in the rod and its maximum temperature. Neglect azimuthal and axial conduction.
- What happens if the rod was made of stainless steel instead (*k* = 18 W/m-K)

Example 2

- A cylindrical element is composed of two zones. The inner zone has a radius of 7/16”, a thermal conductivity of 103 BTU/hr-ft-F, and a volumetric heat generation rate of 1×10^6 BTU/hr-ft^3.The outer zone has a radius of 5/8”, a thermal conductivity of 46.4 BTU/hr-ft-F, and no heat generation. The rod is cooled by convection with the bulk temperature at 400F and a heat transfer coefficient of 1000 BTU/hr-ft^2-F. Neglecting azimuthal and axial conduction, determine the steady state temperature distribution in the element.