Homework 4

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Thursday 20^{th} March, 2025
NUCE 2113

1 Knoll 6.7

A given voltage-sensitive preamplifier requires a minimum input pulse amplitude of 10 mV for good signal-to-noise performance. We have an argon-filled proportional counter of 200 pF (i.e. $2.00 \times 10^{-10}\,\text{F}$*) and wish to detect X-rays of* $50\,\text{keV}$*.*

First, the energy required to produce one ion pair in argon (the "W" value) is taken to be about $26\,\text{eV}$. Thus, for a $50\,\text{keV}$ photon:

$$\text{Number of primary electron-ion pairs} \approx \frac{50{,}000\,\text{eV}}{26\,\text{eV}} \approx 1{,}923 \quad \text{(electrons)}.$$

The total charge Q corresponding to these electrons is:

$$Q = (1{,}923) \times (1.602 \times 10^{-19}\,\text{C}) \approx 3.08 \times 10^{-16}\,\text{C}.$$

If this charge is collected on a capacitor of $200\,\text{pF}$, the resulting pulse amplitude (voltage) without gas multiplication is:

$$V = \frac{Q}{C} = \frac{3.08 \times 10^{-16}\,\text{C}}{2.00 \times 10^{-10}\,\text{F}} \approx 1.54 \times 10^{-6}\,\text{V} = 1.54\,\mu\text{V}.$$

Because the preamplifier needs $10\,\text{mV}$ for acceptable performance, the required gas multiplication factor M must raise $1.54\,\mu\text{V}$ to $10\,\text{mV}$:

$$M = \frac{10\,\text{mV}}{1.54\,\mu\text{V}} \approx 6.5 \times 10^3.$$

Hence, a gas multiplication factor on the order of 6,500 is required.

2 Knoll 4.11

We are told the moon subtends a diameter of about 0.5° *as viewed from Earth. The question asks for the probability that a laser beam, aimed entirely at random from the Earth's surface, will strike the moon.*

For small angles θ in radians, a circular object on the celestial sphere subtends a solid angle:

$$\Omega \approx \pi \left(\frac{\theta}{2}\right)^2 \quad \text{(where } \theta \text{ is the full angular diameter).}$$

Converting 0.5° to radians:

$$0.5^\circ = 0.5 \times \frac{\pi}{180} \approx 8.726 \times 10^{-3}\,\text{rad}.$$

Half of that is 4.363×10^{-3} rad. Thus the solid angle is:

$$\Omega \approx \pi \times (4.363 \times 10^{-3})^2 \approx 6.0 \times 10^{-5}\,\text{sr}.$$

All possible directions in three-dimensional space span 4π sr. Therefore, the probability P that a random direction intersects the moon is:

$$P = \frac{\Omega}{4\pi} \approx \frac{6.0 \times 10^{-5}}{4\pi} \approx 4.8 \times 10^{-6}.$$

In other words, there is roughly a 4.8×10^{-6} (one in about 200,000) chance of hitting the moon if a laser is directed completely at random.