University of Pittsburgh

ME/ENGR 2100 Fundamentals of Nuclear Engineering

Neutron Diffusion:

Neutron Balance

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Related Reading

- Chapter 5 of Duderstadt and Hamilton

OR

- Sections 5.1-5.7 Lamarsh and Baratta
- Sections 6.1-6.4 Lamarsh and Baratta

Learning Objectives

- Define the scalar neutron flux and net neutron current and provide a physical interpretation of each quantity

- Write the mathematical conservation relationship describing the "exact" neutron balance (continuity equation) in non-multiplying systems

Define the scalar neutron flux and net neutron current and provide a physical interpretation of each quantity

Reaction Rates

- Reaction rates are written in terms of macroscopic cross sections and neutron flux
  - Flux is rate at which neutrons pass through a spatial position per unit time
  - Units: [neutrons/cm2/ sec]
- Reaction rate

$$R = \phi\Sigma_t \qquad \frac{\text{neutrons}}{\text{cm}^2\cdot\text{sec}} \times \frac{\text{reactions}}{\text{cm}} = \frac{\text{reactions}}{\text{cm}^3\cdot\text{sec}}$$

- Ultimately reaction rate determines power distribution in fuel

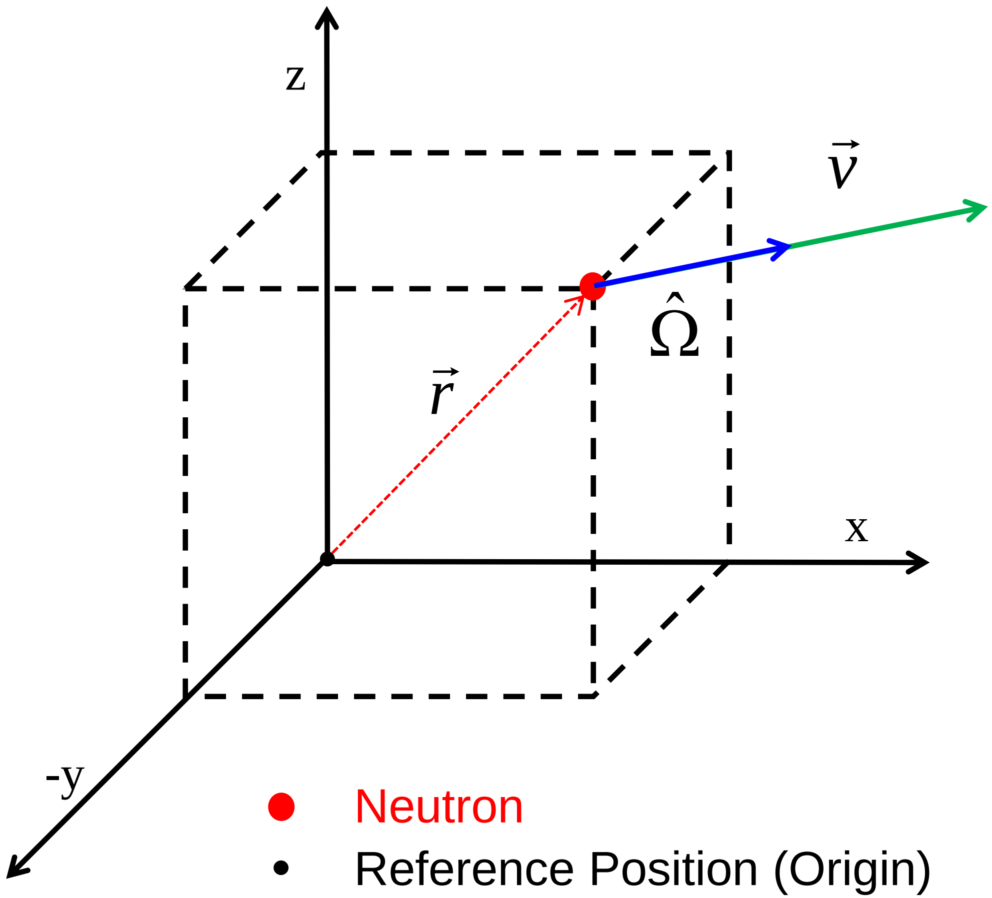
Cross Section Review

- Microscopic
  - Probability per unit area that an incident neutron of given energy and direction will interact with a specific nucleus
  - Units of cm^2 or barns (10^{-24} cm^2)
- Macroscopic
  - Probability per unit length that a specific neutron of known energy and direction will interact
  - Units of 1/cm
- Reaction Rate
  - Rate of neutron reactions (per unit volume and time) in a material
  - Units of [Reactions/cm^3/sec]

Neutron Phase Space

- The location of any neutron is described *uniquely* by 7 independent phase variables
  - Spatial Location: $\left(x, y, z\right) = \vec{r}$
  - Direction of travel: $\hat{\Omega} = \left(v_x, v_y, v_z\right) / \| v \|$
  - Energy (velocity): E
  - Time: t
- These phase space variables are the independent variables in the equation describing the true balance of neutrons in a system
- To simplify the problem we will assume all neutrons have a single energy and that the same number of neutrons are traveling in each direction

Neutron Phase Space



Neutron Density

- Using these assumptions, the neutron density (distribution) at any point in a reactor is

$$n(\vec{r},t)\,dV$$

- This gives the number of neutrons that fall within volume dV at time t.
- Referred to as the *Neutron Density*.
- Sometimes referred to as scalar neutron density to indicate it includes neutrons traveling all directions

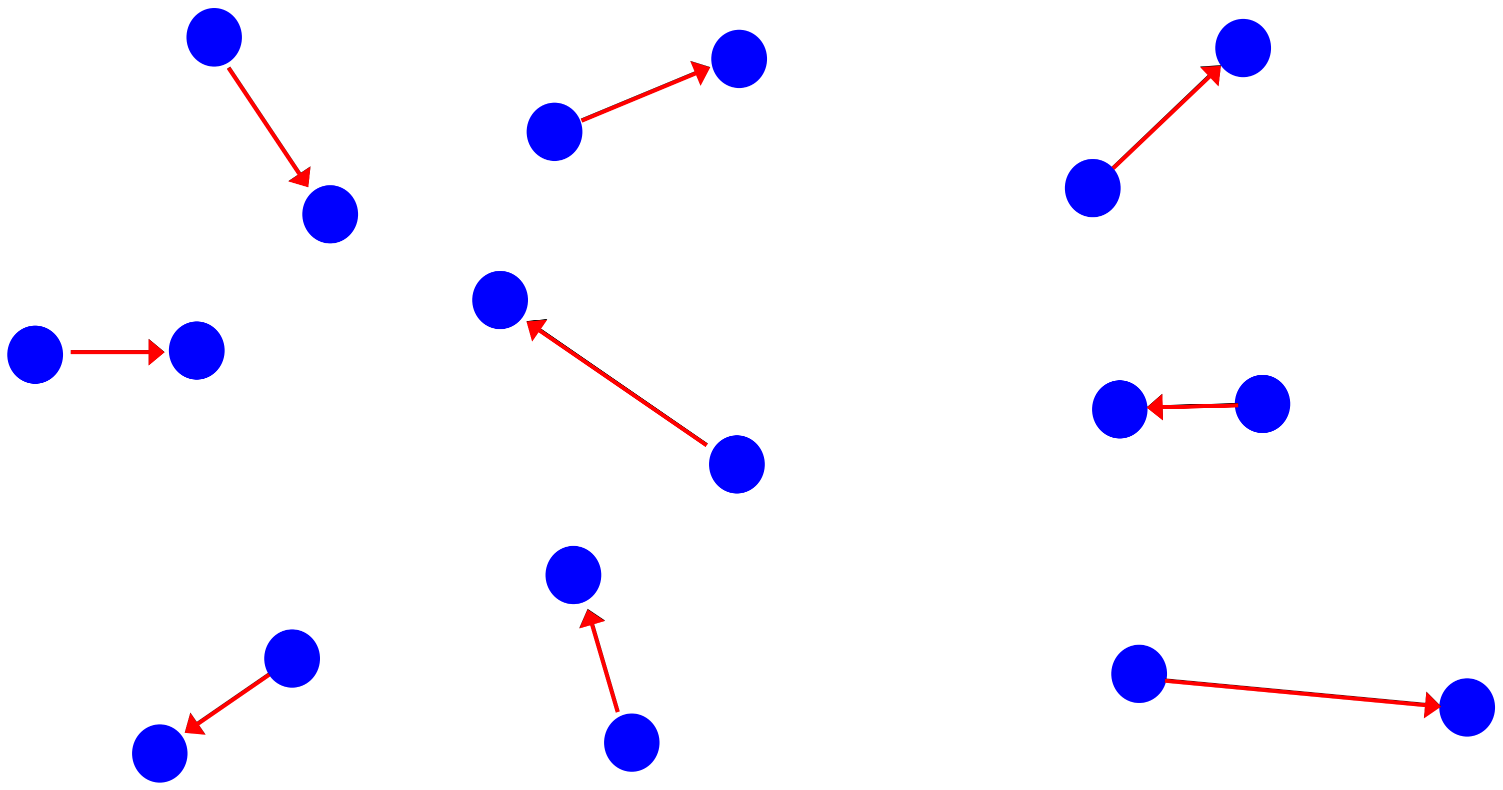
Neutron Flux

- Neutron Velocity: $\mathrm{v} = \sqrt{2E/m_n}$ [length/time]
- Velocity × Time = Distance Traveled
  v × dt = ds
- ds is the distance traveled by one neutron in dt.

$$dS = \mathrm{v}\, dt\, n(\vec{r},t)\, dV$$

- dS is the total distance that neutrons in dV about $(\vec{r})$ travel during time dt.

Neutrons at $t = dt$



Total distance traveled by all neutrons during dt
= Total path length generated by all neutrons during dt

Scalar Neutron Flux

- dS is the total path length generated by all neutrons in dV about $(\vec{r})$ during time interval dt.
- Dividing dS by dt gives

  $$\phi(\vec{r},t)\,dV = \mathrm{v}n(\vec{r},t)\,dV = \frac{dS}{dt}$$

  the rate at which neutrons in dV about point $(\vec{r})$ generate path length at time t.

- $\phi(\vec{r},t)$ is referred to as the <u>scalar neutron flux</u>

Reaction Rate

$$R_t(\vec{r},t) = \Sigma_t(\vec{r},t)\phi(\vec{r},t)$$

- Total Reaction Rate Density
  - Rate at which neutrons at position $\vec{r}$, undergo any type of reaction.

$$R_x(\vec{r},t) = \Sigma_x(\vec{r},t)\phi(\vec{r},t)$$

- Reaction Rate Density
  - Rate at which neutrons at position $\vec{r}$, undergo reaction type *x*.

Write the mathematical conservation relationship describing the “exact” neutron balance (continuity equation) in non-multiplying systems

The equation of continuity

- The equation of continuity is the statement of the fact that since neutrons do not disappear unaccountably, the time rate of change in the number of neutrons in volume V must be accounted for by the known physical processes of production, absorption, and leakage

$$\begin{bmatrix} \text{Rate of change of} \\ \text{number of} \\ \text{neutrons in V} \end{bmatrix} = \begin{bmatrix} \text{Rate of} \\ \text{production of} \\ \text{neutrons in V} \end{bmatrix} - \begin{bmatrix} \text{Rate of} \\ \text{absorption of} \\ \text{neutrons in V} \end{bmatrix} - \begin{bmatrix} \text{Rate of} \\ \text{leakage of} \\ \text{neutrons in V} \end{bmatrix}$$

Rate of change of number of neutrons

- If $n(\vec{r},t)$ is the density f neutrons at any point and time in V the total number of neutrons in V is
  $$\int_V n(\vec{r},t)dV$$
- The rate of change of the number of neutrons in volume V is then
  $$\frac{d}{dt}\int_V n(\vec{r},t)dV = \int_V \frac{\partial n(\vec{r},t)}{\partial t}dV$$

Production rate of neutrons

- In a reactor neutrons are generally produced through fission. To simplify things are first we imagine that we have a well-defined neutron source emitting neutrons in our problem, $s(\vec{r},t)$, at a rate s neutrons per cc per s at point r.
- This is referred to as a non-multiplying or fixed-source problem.
- The rate at which neutrons are produced by this source is

$$\int_V s(\vec{r},t)dV$$

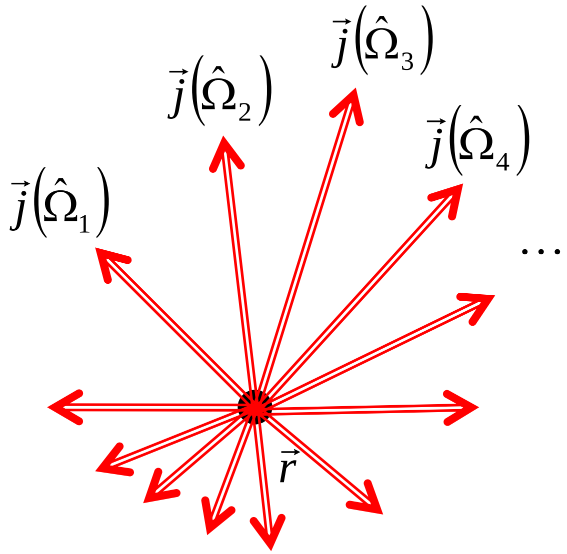
Absorption rate of neutrons

- The rate at which neutrons are lost to absorption (per cc per second) is equal to the neutron absorption rate, which we know can be written $\Sigma_a(\vec{r},t)\phi(\vec{r},t)$
- The rate at which neutrons are absorbed in the volume V is then

$$\int_V \Sigma_a(\vec{r},t)\phi(\vec{r},t)dV$$

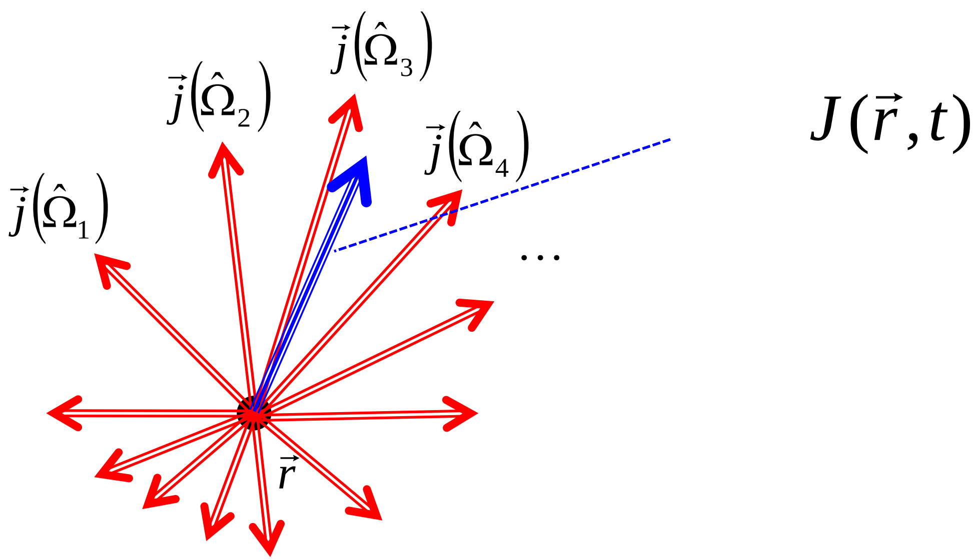
Leakage rate of neutrons

- Describing the leakage rate of neutrons requires the introduction of an additional concept called the neutron current
- Imagine we knew the neutron flux in individual directions and used the symbol j to describe the flux in each direction in vector form. j is called the neutron current



Neutron Current

- Many of the vectors cancel each other out, leaving us with a vector that represents the net direction and rate of neutron flow through $\vec{r}$.



Neutron Current

- **Neutron Current Density** $\vec{J}\left(\vec{r},t\right)$
  - A vector pointing in the direction of net neutron flow, whose magnitude gives the net rate at which neutrons are passing through point $\vec{r}$ at time t.

- If J is the neutron current density vector on the surface of V and **n** is a unit normal pointing outward from the surface then

$$J(\vec{r},t)\cdot n$$

- is the number of neutrons passing outward through the surface per square cm per second.

Leakage rate of neutrons

- The total rate at which neutrons are lost due to leakage is the sum of the leakages at each point on the surface (integral over surface area)

$$\int_A \left(J(\vec{r},t) \bullet n \right) dA$$

- Using the divergence theorem this can also be written as a volume integral

$$\int_A \left(J(\vec{r},t) \bullet n \right) dA = \int_V \left(\nabla \bullet J(\vec{r},t) \right) dV$$

The equation of continuity

- The *equation of continuity* can be written

$$\int_V \frac{\partial n(\vec{r},t)}{\partial t} dV = \int_V s(\vec{r},t)dV - \int_V \Sigma_a(\vec{r},t)\phi(\vec{r},t)dV - \int_V \left(\nabla \bullet J(\vec{r},t)\right) dV$$

- Because all of the integrals were carried out over the same arbitrary volume the integrands must be equal. Thus

$$\frac{\partial n(\vec{r},t)}{\partial t} = s(\vec{r},t) - \Sigma_a(\vec{r},t)\phi(\vec{r},t) - \nabla \bullet J(\vec{r},t)$$

The equation of continuity

- Using our definition of scalar flux we can rewrite the time rate of change term:

$$\frac{1}{v}\frac{\partial \phi(\vec{r},t)}{\partial t} = s(\vec{r},t) - \Sigma_a(\vec{r},t)\phi(\vec{r},t) - \nabla \bullet J(\vec{r},t)$$

- Unfortunately we still have an equation with two unknowns, neutron flux and neutron current!

$$\frac{1}{v}\frac{\partial \phi(\vec{r},t)}{\partial t} = s(\vec{r},t) - \Sigma_a(\vec{r},t)\underline{\phi(\vec{r},t)} - \nabla \bullet \underline{J(\vec{r},t)}$$