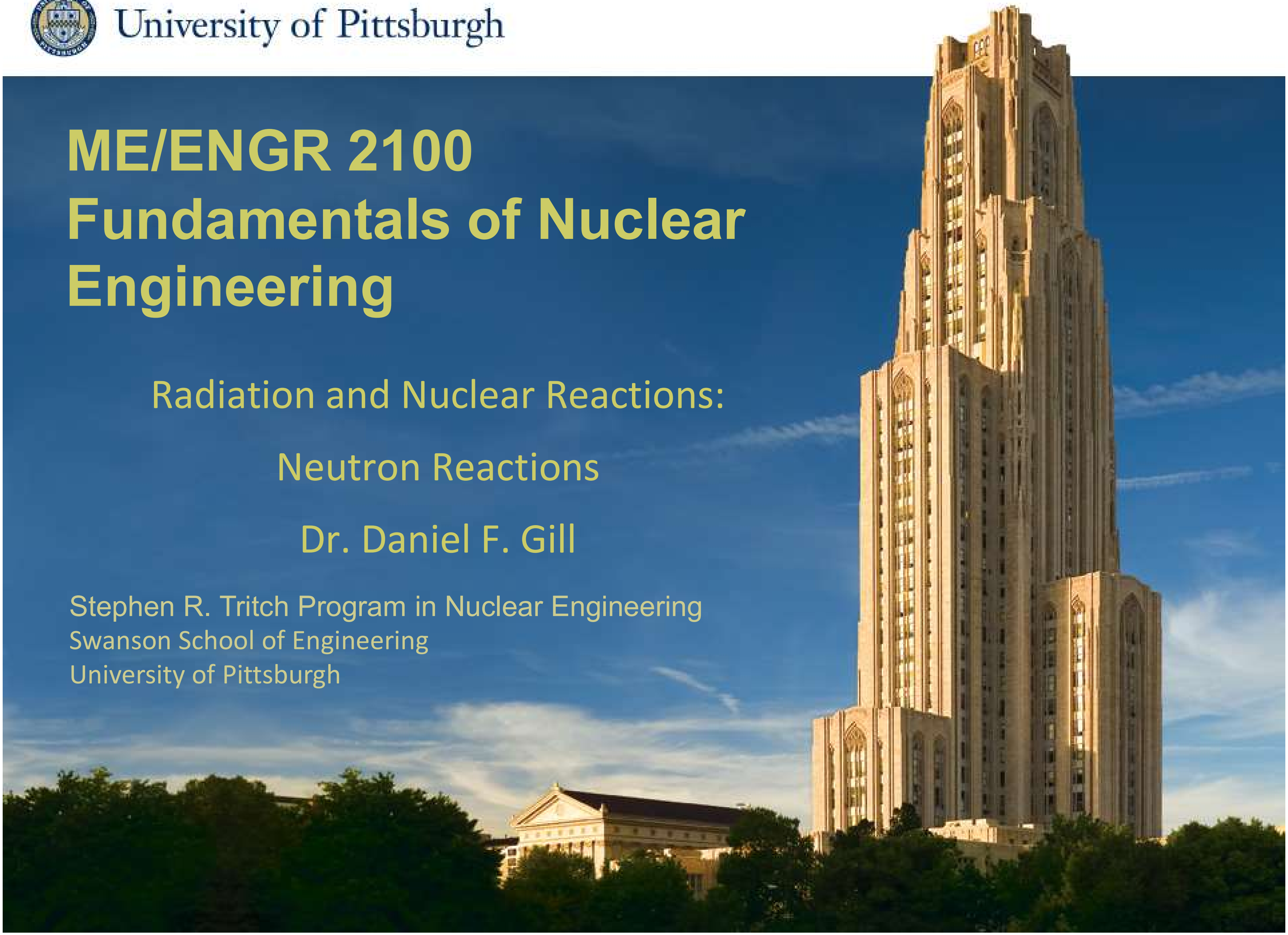
University of Pittsburgh

ME/ENGR 2100 Fundamentals of Nuclear Engineering



Radiation and Nuclear Reactions:

Neutron Reactions

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Relevant Reading Assignments

- Chapter 2/3 of "Introduction to Nuclear Engineering," Lamarsh and Baratta, 3rd edition, Prentice-Hall (2001)
- Chapter 2 of "Nuclear Engineering: Theory and Technology of Commercial Nuclear Power," Knief, 2nd edition, American Nuclear Society (1992, reprint by ANS 2008)
- Chapter 2 of "Nuclear Reactor Analysis," Duderstadt and Hamilton, Van Nostrand (1976)
- Module 1 of DOE Fundamentals Handbook, "Nuclear Physics and Reactor Theory," U.S.DOE (1993)Available at:

https://www.standards.doe.gov/standards-documents/1000/1019-bhdbk-1993-v1

- Not required but useful and clear is the discussion of nuclear masses and binding energies at the beginning of Chapter 7 of "Concepts of Nuclear Physics" by Bernard L. Cohen, McGraw-Hill, 1971, available in most scientific libraries.

Learning Objectives

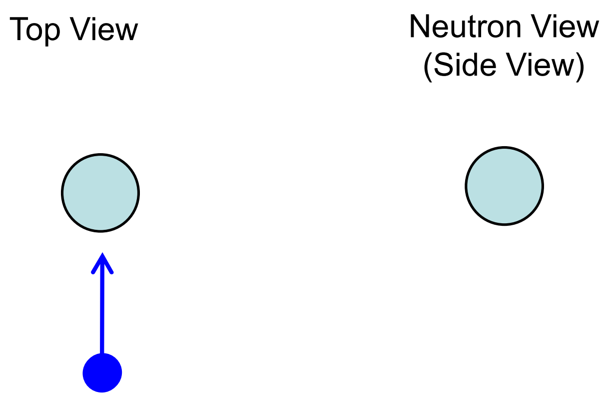
- Define microscopic cross sections. Sketch the first three of the four tiers in the cross section hierarchy.
- Define macroscopic cross section and mean free path.
- Define neutron flux and explain the equation for neutron reaction rate.
- Interpret the energy dependencies of neutron-reaction cross sections.

Define microscopic cross sections. Sketch the first three of the four tiers in the cross section hierarchy

Nuclear Reaction Rates

- So far we have covered:
  - Atomic / nuclear structure
  - Nuclear stability
  - Types of radioactive decay / radiation
  - Radiation interactions
  - Nuclear interactions
- Remaining question:
  - How can we predict the expected frequency of interactions in a given radiation field?

Probability of an Interaction – the microscopic cross-section



- We start by considering the probability that a neutron will strike a single nucleus
  - Assume that the neutron and nucleus are both solid spheres (or a plate on a barn wall).
  - The neutron "sees" the nucleus as a round target with an effective area, viz., the microscopic cross-section.
  - Microscopic cross-sections are defined with units of area, and are proportional to the probability of the interaction occurring.

- Atom
- Free Neutron

Microscopic Cross Section

- Microscopic cross section
  - Cross sectional area of nucleus as seen by neutron, denoted by symbol σ
  - Has units of area, given in units of barns
    - 1 barn = 10−24 cm2
  - Proportional to the probability that a neutron will strike the nucleus and undergo a reaction
  - Nuclide dependent

Microscopic Cross Section is not just a simple addition of radii

Microscopic Cross Sections – Calculated vs. Measured

Neutron radius: $r_n = 0.85 \times 10^{-15}$ [meters]

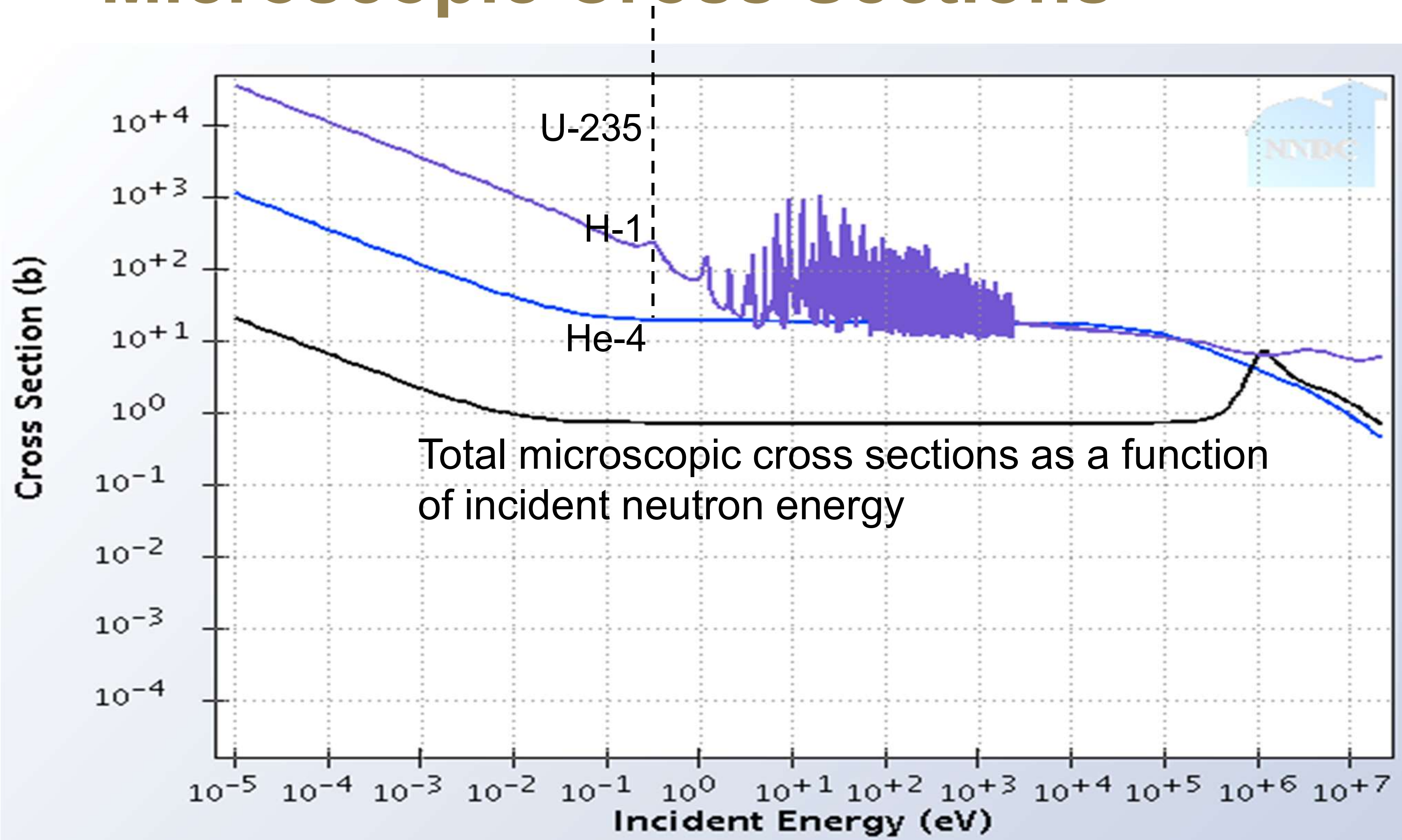
Nuclear radius: $r_a = 1.2 \times 10^{-15} (A)^{1/3}$ [meters] (A is atomic mass)

Guessed Microscopic Cross Section: $\sigma = \pi(r_n + r_a)^2$ [meters^2]

Nuclide	Guessed σ	Measured σ
^{4}He	0.132 barns	0.759 barns
^{1}H	0.238 barns	20.43 barns
^{235}U	2.140 barns	~500 barns

- So a solid spheres model is not a good approximation

Microscopic Cross Sections



Microscopic Cross Section

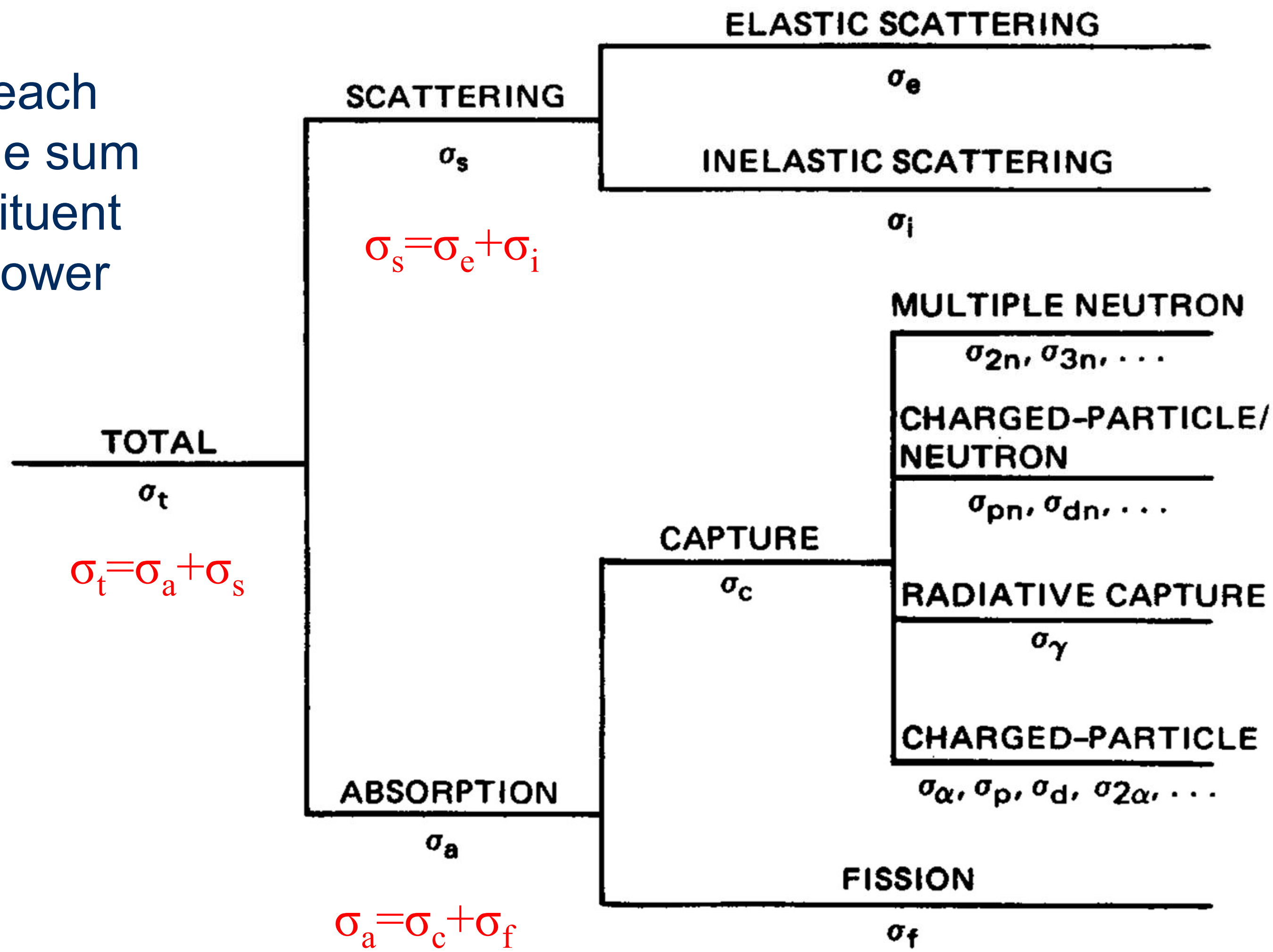
- The microscopic cross section depends heavily on:
  - The structure / stability of the target nucleus
    - Partially filled neutron shells are more receptive to a neutron interaction than completely filled shells
  - The energy of the neutron
    - Typically low-energy (slow) neutrons are more likely to interact with a nucleus than high-energy (fast) neutrons.
    - <u>The neutron energy dependence is extremely complicated</u>

Microscopic Cross Section

- So far we have only considered the probability that *any type* of reaction will occur:
  - This is referred to as the Total Microscopic Cross Section, σ_t
- We can also consider the probability that a *specific type* of reaction will occur:
  - Microscopic Scattering Cross Section, σ_s
  - Microscopic Absorption Cross Section, σ_a
    - Microscopic Capture (n,γ) Cross Section, σ_c
    - Microscopic Fission Cross Section, σ_f

Cross Section Hierarchy

Micros on each level are the sum of all constituent micros on lower levels.



Define macroscopic cross section and mean free path

Macroscopic Cross Sections

- Microscopic nuclear cross sections (σ) describe the probability of a particular reaction occurring with a particular nuclide (e.g., U-235, O-16, other) for a neutron of a specific energy. The unit of cross-section is barns (10-24cm^2).
- Since bulk behavior in the material medium depends on how many nuclei of interest are present, we consider the product of σ and the number density (number x cm-3) of those nuclei.
- This product, termed the macroscopic cross section, is labeled with the capitalized Greek letter sigma: Σ (with units of cm^{-1}).
- This can be thought of as the inverse of the average distance traveled by a neutron in the material before this reaction occurs.

Macroscopic Cross Sections

- So to form Macroscopic cross sections we multiply the probability of interacting with a single nucleus by the number (density) of nuclei in the material.
  - The quantity Σ is called the macroscopic cross section, it has units of 1/cm.

$$\boxed{\Sigma_t = N\sigma_t}$$

Units: [nuclei/cm^3] x [cm^2/nucleus]

  - These will be used subsequently as we learn to calculate reaction rates.

Macroscopic Cross Sections

- The macroscopic cross section gives the probability that a neutron will undergo a reaction per distance travelled (1/cm).
  - Since microscopic cross sections are energy dependent it follows that macroscopic cross sections are as well.
  - Macroscopic cross sections for individual reaction types can be calculated from the corresponding microscopic cross sections.

$$\Sigma_a = N\ \sigma_a \qquad \Sigma_s = N\ \sigma_s \qquad \Sigma_f = N\ \sigma_f$$

Mean Free Path λ

- The mean free path is then defined as

$$\lambda = \frac{1}{\Sigma}$$, having units of [cm]

  - If a neutron beam is traveling through a material and exponentially decreasing in intensity with distance, this quantity is the attentuation constant, i.e.,

$$\phi(x) = \phi_0 \exp(-\frac{x}{\lambda})$$

Macroscopic Cross Sections

$$N^j \sigma_r^j = \Sigma_r^j(E)$$

- Single nuclide, single interaction: $N\sigma_r = \Sigma_r$
- Single Nuclide, multiple interactions

$$\Sigma_t = N(\sigma_c + \sigma_f + \sigma_s) = \Sigma_c + \Sigma_f + \Sigma_s$$

$$\Sigma_t = \sum_{all\, i} N\sigma_i = \sum_{all\, i} \Sigma_i$$

- Multiple nuclides, multiple interactions

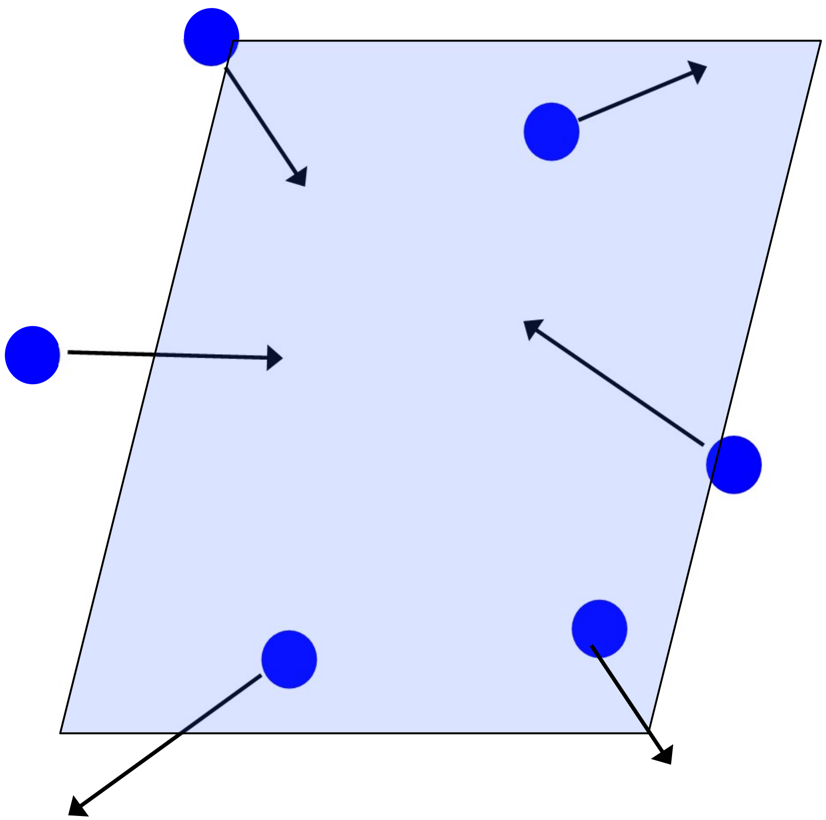
$$\Sigma_t^{mix} = \sum_{all\ j} \sum_{all\, r} \Sigma_r^j = \sum_{all\ j} \sum_{all\, r} N^j \sigma_r^j$$

Define neutron flux and explain the equation for neutron reaction rate

Scalar Neutron Flux

- $\phi(r,t)$represents the *neutron flux* as a function of position and time.
- "Flux" in physics represents generally the number of entities (neutrons, photons, magnetic field lines, raindrops, other…) impinging on a unit area of a system per unit time.
- Neutron flux is calculated as the number density of neutrons multiplied by their speed, i.e., :$\phi = nv$, where n = number of neutrons per cc and v is their speed in cm/sec. Hence the unit of flux is [neutrons / cm^2 / sec], representing the number of neutrons passing through a given area per unit time.

What is Neutron Flux?

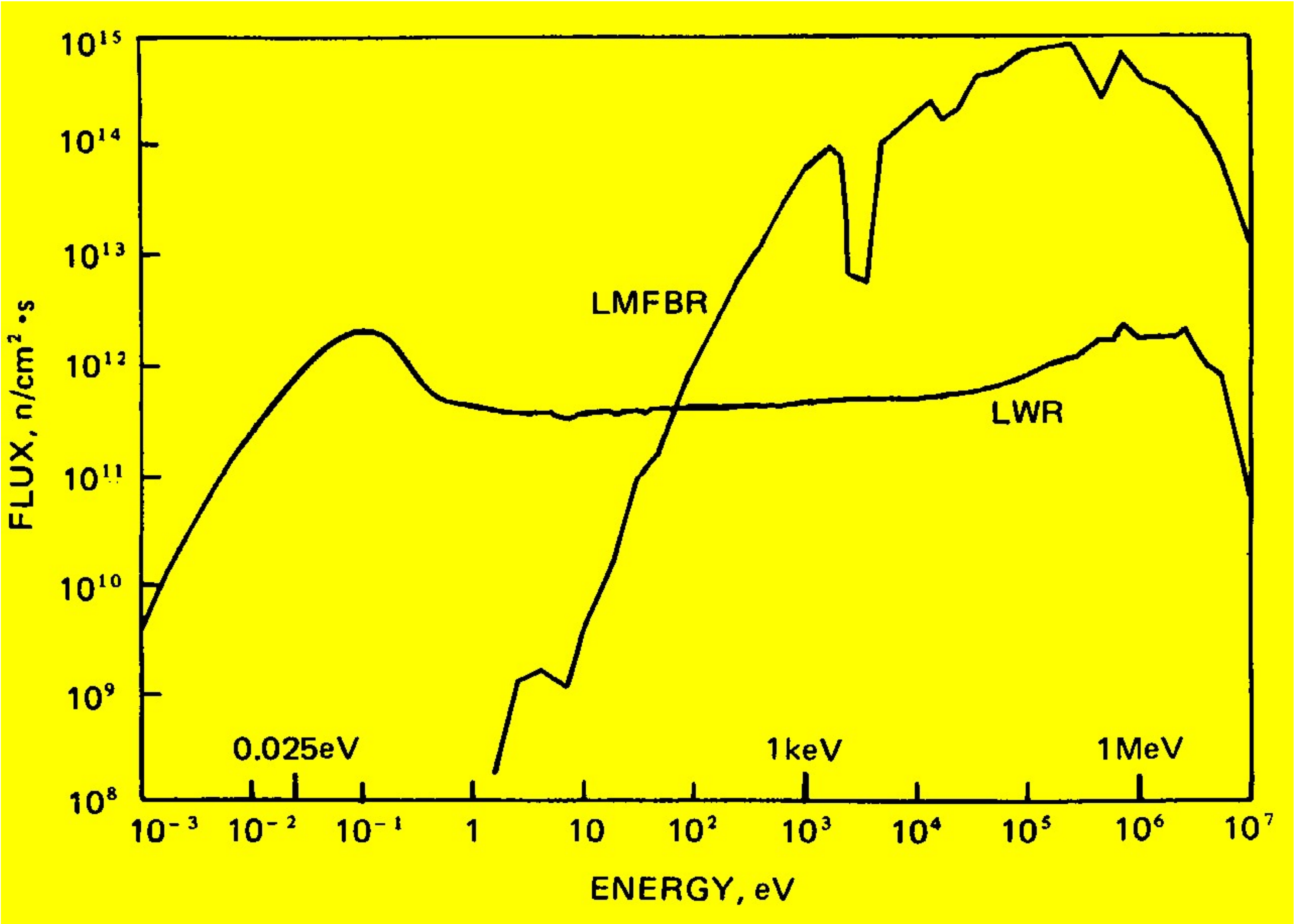


Number of neutrons impinging on a unit area of a system per unit time, travelling in any direction. Units are [1/cm^2-sec]

We will now define the reaction rate concept.

Neutron Fluxes

Flux spectrum (energy dependence)



Reaction rates: definition and utility

Reaction Rate definition: a rate of a particular reaction occurring per second, per unit volume, in some physical region of interest.

- Of course, the fission rate per unit volume within the fuel pellet gives rise to the heat source in the nuclear reactor core
- Reaction rates utilize microscopic nuclear cross sections (σ), which, in units of area, describe the probability of a particular reaction occurring with a particular nuclide (e.g., U-235, O-16, other) for a neutron of a specific energy. The unit of cross-section is barns ($10^{-24} cm^2$).
- Since bulk behavior in the material medium depends on how many nuclei of interest are present, we consider the product of σ and the number density (number x cm^{-3}) of those nuclei. This product, termed the macroscopic cross section, is labeled with the capitalized Greek letter sigma: Σ (with units of cm^{-1}). This can be thought of as the inverse of the average distance traveled by a neutron in the material before this reaction occurs.

Reaction Rates

- The rate of (all) neutron interactions (per unit volume) in a material is given by:

$$\boxed{R = \Phi \Sigma_t}$$

Units: [neutrons/cm^2/sec] × [reactions/cm]
= [reactions/cm^3/sec]

  - The rate of any individual reaction can be calculated by substituting the individual reaction macroscopic cross section for the total macroscopic cross section shown above.

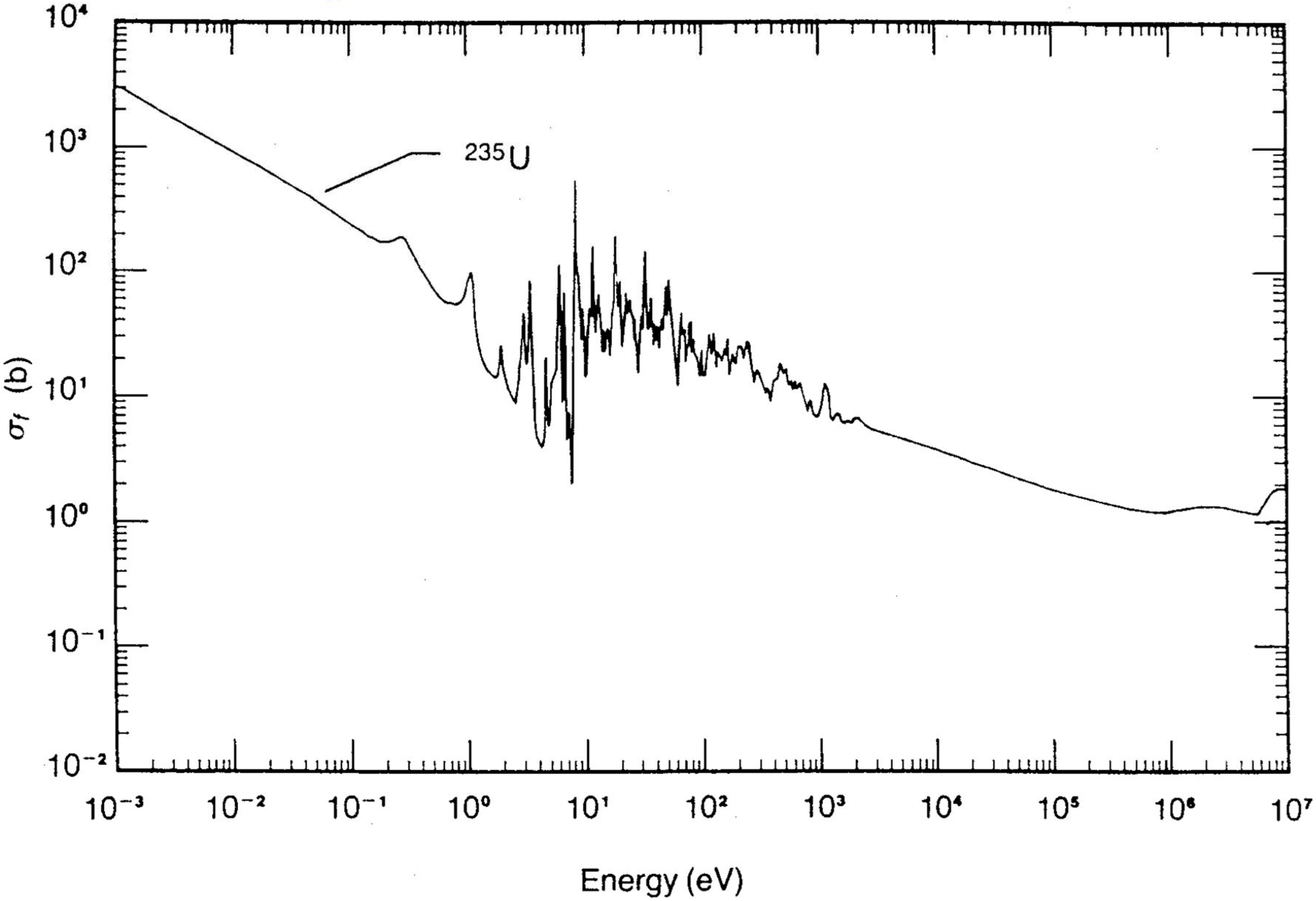
Reaction Rate:

$$R_x\left(\vec{r},t\right)=\int_0^\infty \Sigma_x\left(\vec{r},E,t\right)\Phi\left(\vec{r},E,t\right)dE$$

- Reaction Rate Density (Energy Integrated)
  - Rate at which neutrons at position $\vec{r}$, **all energies**, undergo a reaction of type x.
  - This reaction rate density is what must be calculated to design a reactor; it governs where the fission energy is deposited.

Interpret the energy dependencies of neutron-reaction cross sections

Microscopic Cross Section

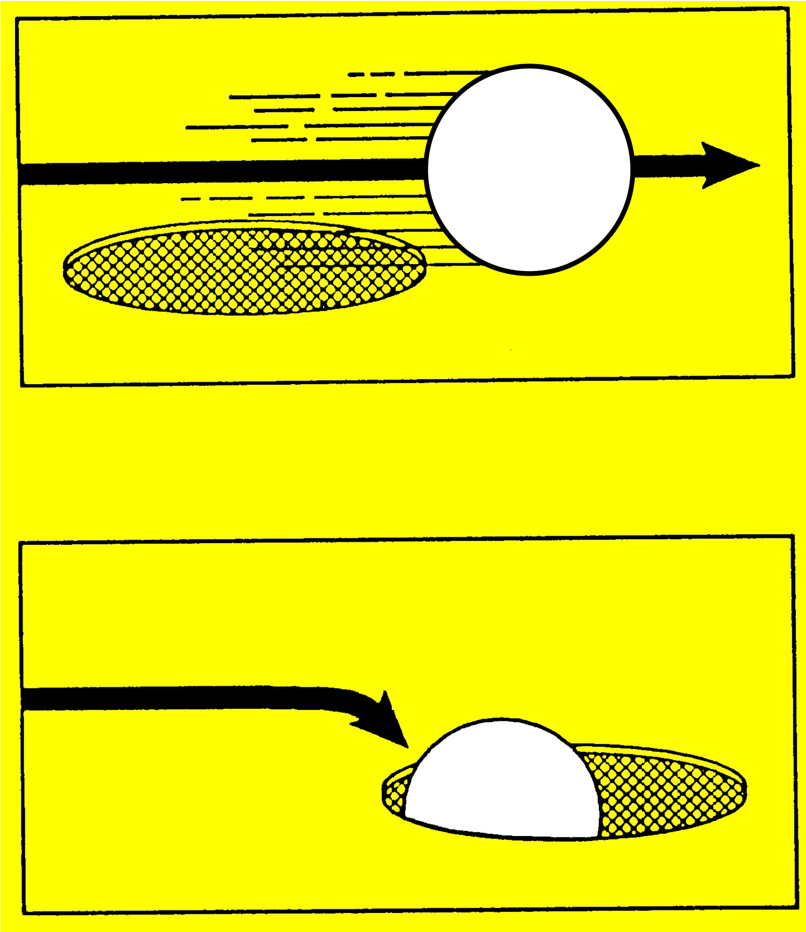


Microscopic Cross Section

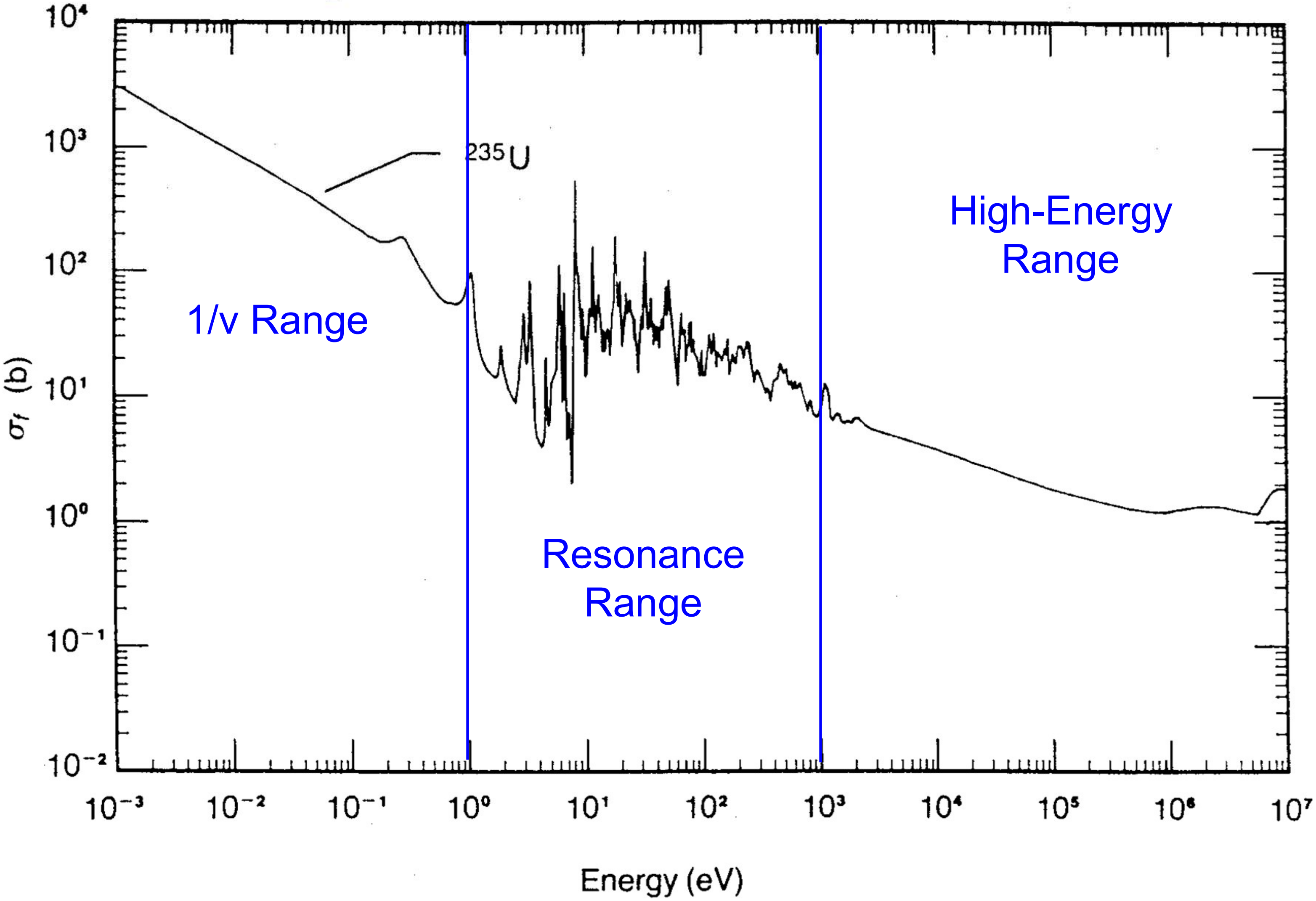
- The energy dependence of microscopic cross sections can be divided into three ranges:
- **High Energy** (neutron energy > 1 keV)
  - Quantum effects are less important and probability of interaction shows little variation.
- **Resonance Range** (1 eV < neutron energy < 1 keV)
  - Quantum effects dominate and probability of interaction depends on how closely the neutron energy matches an unfilled nuclear shell in the target nuclide.
- **Thermal / 1-over-v Range** (neutron energy < 1 eV)
  - Probability of interaction increases as neutron energy (velocity) decreases

$$\sigma(E) = \frac{\sigma_0 \mathrm{v}_0}{\mathrm{v}} = \sigma_0 \sqrt{\frac{E_0}{E}}$$

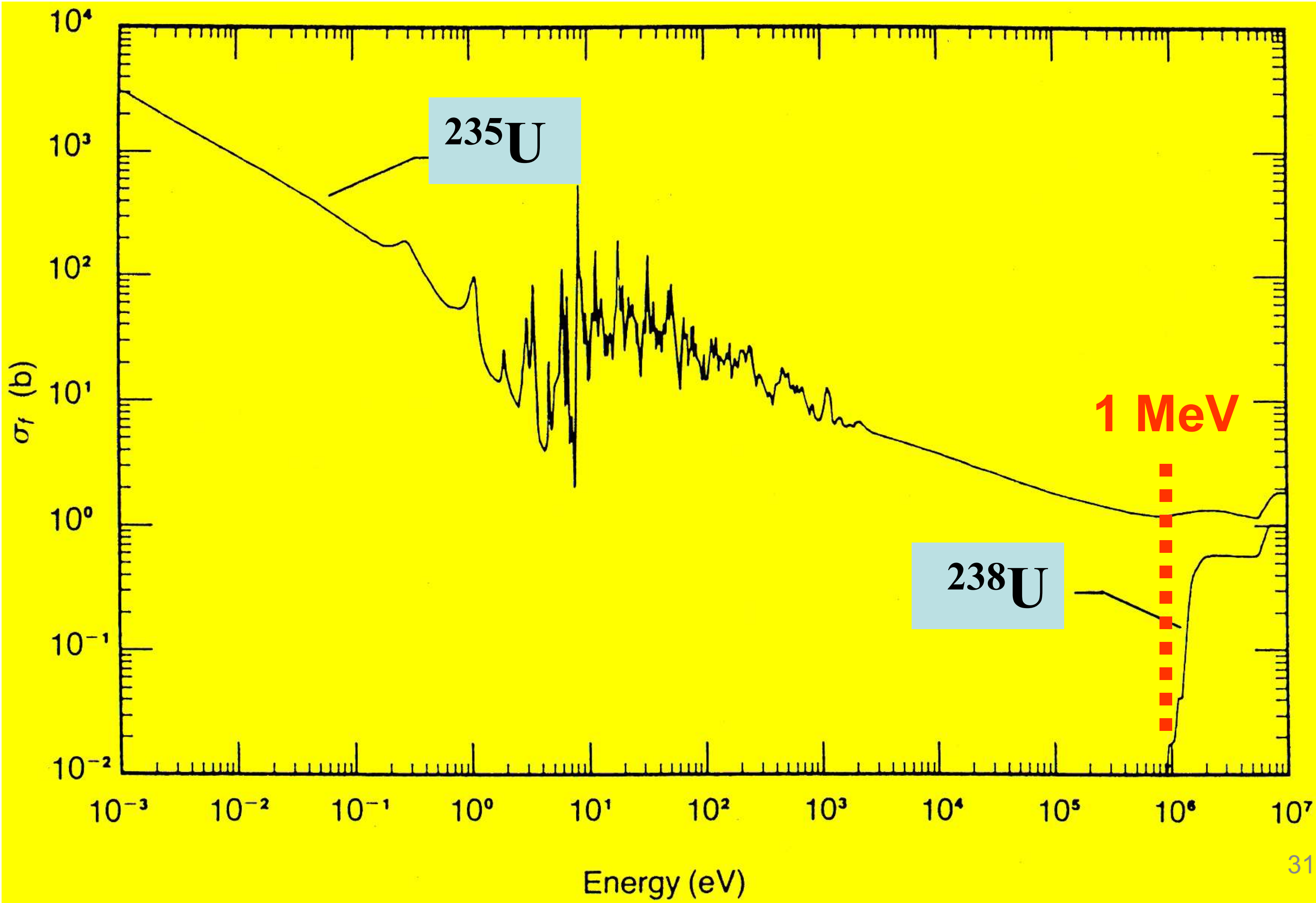
Analogy for “1-over-v” Neutron Absorption

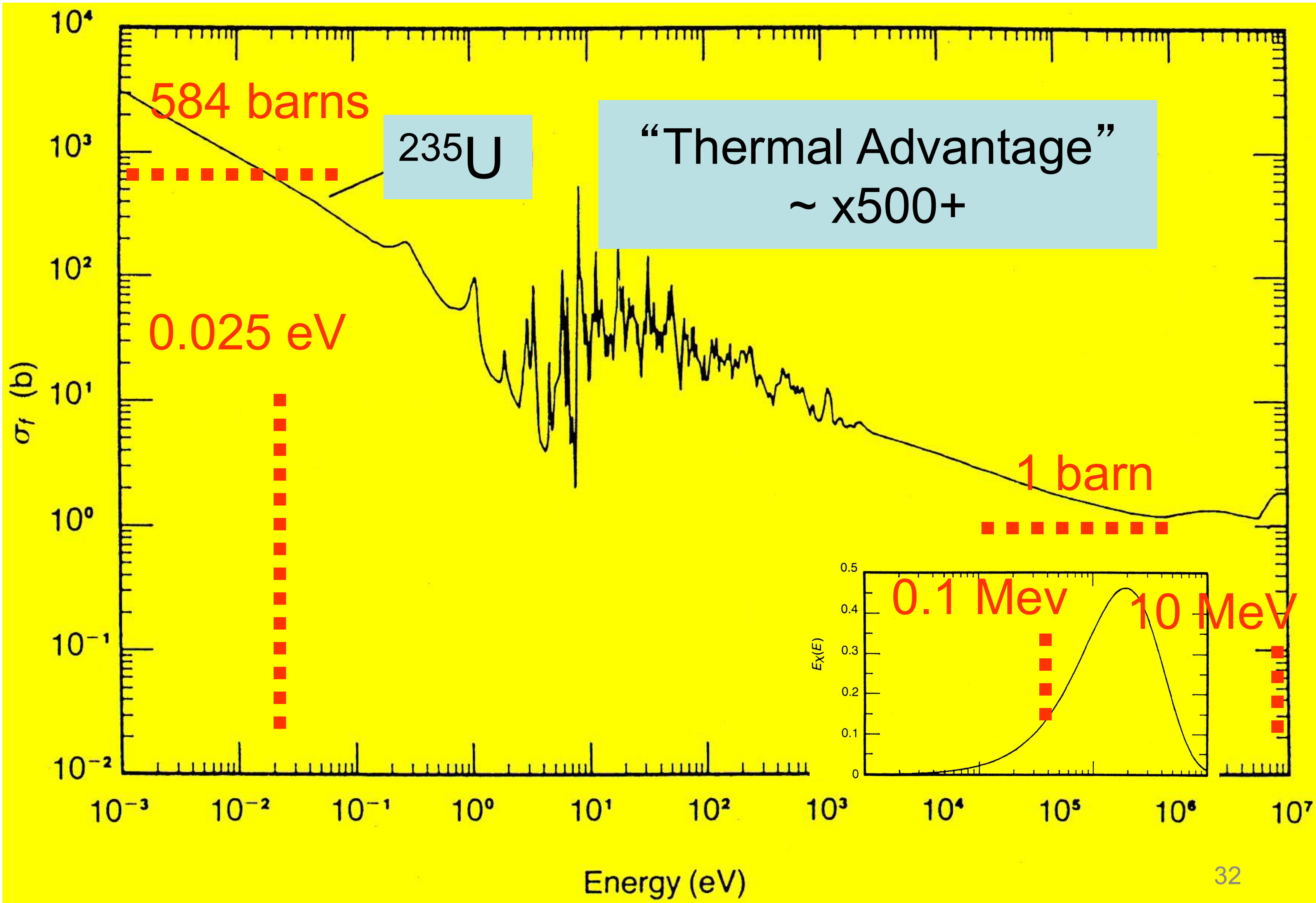


Microscopic Cross Section

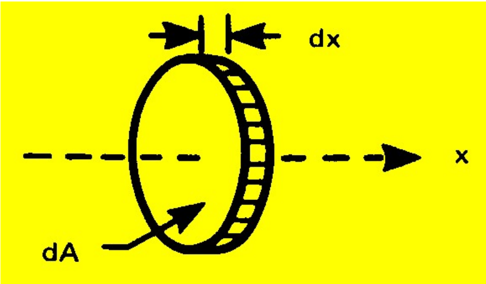


Uranium Fission Cross Sections




584 barns
235U
"Thermal Advantage"
~ x500+
0.025 eV
1 barn
0.1 Mev
10 MeV
σf (b)
Energy (eV)
10⁻³
10⁻²
10⁻¹
10⁰
10¹
10²
10³
10⁴
10⁵
10⁶
10⁷
Eχ(E)

Interaction Rate



- Reaction Rate
  - Reaction Rate = (n v dA) (N σ dx)
  - Reaction Rate = (n v) (N σ) dV
  - Reaction Rate = Φ Σ dV
  - Neutron Flux = Φ (neutrons/cm2-s) = n v
  - Macroscopic Cross Section = Σ (cm-1) = N σ
  - Reaction Rate = N σ Φ = Φ Σ per Unit Volume

Neutron Beam Attenuation

- Neutron Beam Interaction

$$-d\Phi(x) = \Sigma\,\Phi(x)\,dx$$

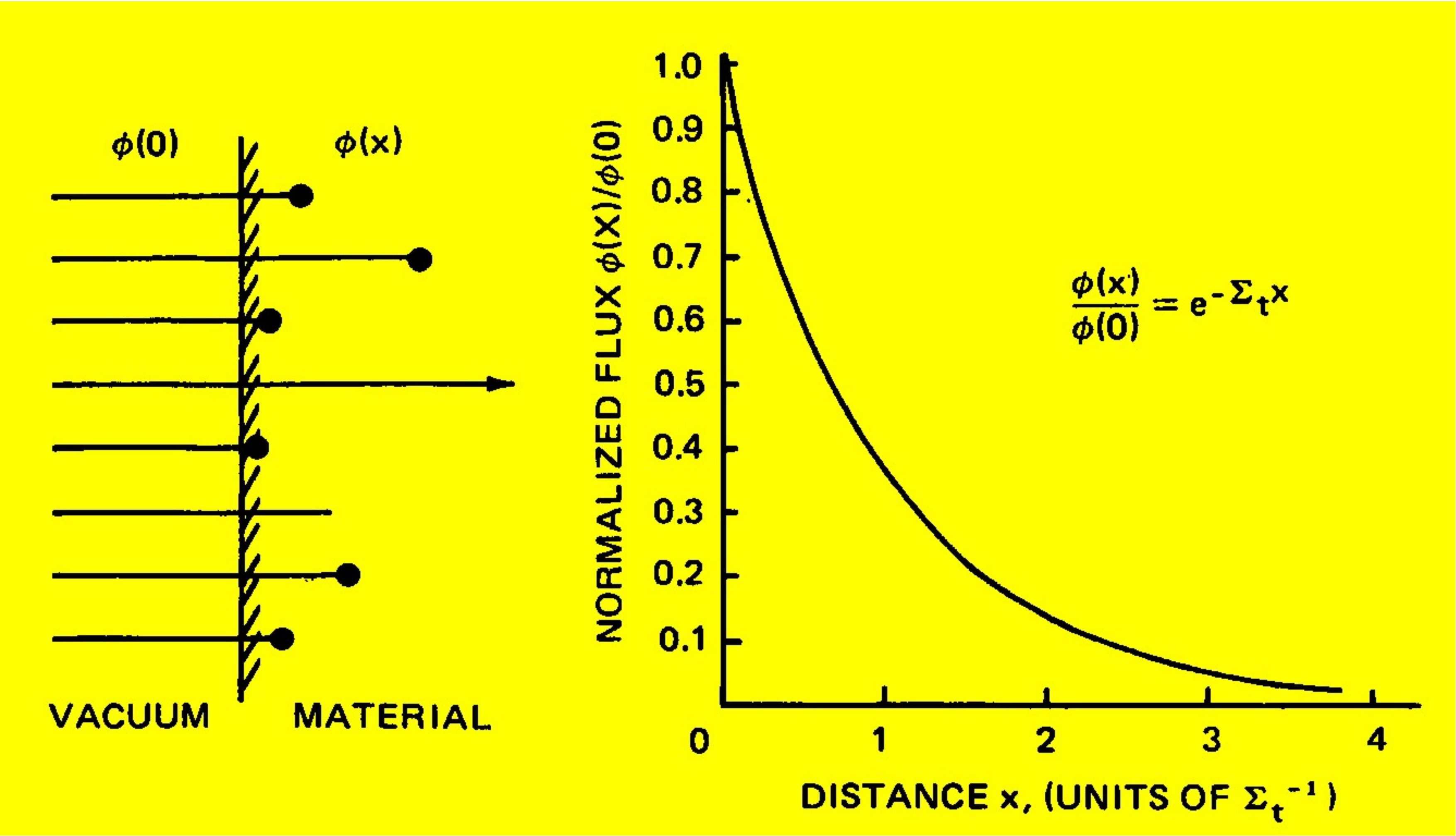
$$\Phi(x) = \Phi(0)\,e^{-\Sigma x}$$

$$\frac{\Phi(x)}{\Phi_0} = e^{-\Sigma x}$$

- Thus macroscopic cross section

$$\Sigma = \frac{-d\Phi(x)}{\Phi(x)\,dx} = \frac{-d\Phi(x)/\Phi(x)}{dx}$$

Neutron Attenuation



Example Problem 1

A material has a microscopic neutron cross section of 3.5 barns, and contains and contains 4.2×10^{23} nuclei/cm^3. What is:

The macroscopic cross section?

The mean free path?

Example Problem 1

A material has a microscopic neutron cross section of 3.5 barns, and contains and contains 4.2 x 10^{23} nuclei/cm^3. What is:

The macroscopic cross section?

$$\sigma_t = 3.5 barns = 3.5x10^{-24}\ cm^2 / nucleus;\ N = 4.2x10^{23} nuclei / cm^3$$

$$Total\ macroscopic\ cross\ section =$$

$$\Sigma_t = \left(3.5x10^{-24}\ cm^2 / nucleus\right)\left(4.2x10^{23} nuclei / cm^3\right) = 1.47\,cm^{-1}$$

The mean free path?

$$\lambda = \frac{1}{\Sigma_t} = \frac{1}{1.47\,cm^{-1}} = 0.68\,cm$$

Example Problem 2

A boiling water reactor operates at 1000 psi. At that pressure the density of water and of steam are, respectively, 0.74 g/cm3 and 0.036 g/cm3. The microscopic total cross sections of H and O for thermal energy neutrons are 38 barns and 4.2 barns.

What is the total macroscopic cross section of the water?

What is the total macroscopic cross section of the steam?

If on average, 40% of the volume is occupied by steam, what is the total macroscopic cross section of the steam-water mixture?

Example Problem 2

A boiling water reactor operates at 1000 psi. At that pressure the density of water and of steam are, respectively, 0.74 g/cm3 and 0.036 g/cm3. The microscopic total cross sections of H and O for thermal energy neutrons are 38 barns and 4.2 barns.

What is the total macroscopic cross section of the water?

$$N_H = 2 * \frac{0.74\left(g / cm^3\right) N_A\left(nuclei / mole\right)}{18.015 g / mole} = 4.947 \times 10^{22} (nuclei / cm^3)$$

$$N_O = 1 * \frac{0.74\left(g / cm^3\right) N_A\left(nuclei / mole\right)}{18.015 g / mole} = 2.474 \times 10^{22} (nuclei / cm^3)$$

$$\Sigma_t^{water} = N_H \sigma_H + N_O \sigma_O = 4.947 \times 10^{22} * 38 \times 10^{-24} + 2.474 \times 10^{22} * 4.2 \times 10^{-24} = 1.99 cm^{-1}$$

What is the total macroscopic cross section of the steam?

$$\Sigma_t^{steam} = 2 * \frac{0.036 * N_A}{18.015 g} * 38 \times 10^{-24} + 1 * \frac{0.036 * N_A}{18.015 g} * 4.2 \times 10^{-24} = 0.0966 cm^{-1}$$

If on average, 40% of the volume is occupied by steam, what is the total macroscopic cross section of the steam-water mixture?

$$\Sigma_t^{total} = 0.4\left(0.0966 cm^{-1}\right) + 0.6\left(1.99 cm^{-1}\right) = 1.23 cm^{-1}$$

Example Problem 3

What is the total macroscopic cross section of uranium dioxide (UO2) that has been enriched to 4 at%? Assume

$$\sigma_t^{U235} = 607.5b, \sigma_t^{U238} = 11.8b, \sigma_t^{O} = 3.8b, and\ a\ UO2\ density\ of\ 10.5g / cm^3$$

$$\frac{1}{A_U} = \frac{1}{100}\left(\frac{4}{235.0439} + \frac{96}{238.0508}\right), \qquad A_U = 237.929$$

$$A_{UO2} = 237.929 + 2*15.9994 = 269.9278$$

$$w/o(U) = \frac{237.929}{269.9278}*100 = 88.145$$

$$N_{U235} = \frac{0.04*.88145*10.5\frac{\text{g}}{\text{cm}^3}*0.6022x10^{24}\frac{\text{atoms}}{\text{mol}}}{235.0439\frac{\text{g}}{\text{mol}}} = 9.485\times10^{20}\frac{\text{atoms}}{\text{cm}^3}$$

$$N_{U238} = \frac{0.96*.88145*10.5\frac{\text{g}}{\text{cm}^3}*0.6022x10^{24}\frac{\text{atoms}}{\text{mol}}}{238.0508\frac{\text{g}}{\text{mol}}} = 2.248\times10^{22}\frac{\text{atoms}}{\text{cm}^3}$$

Example Problem 3

$$N_O = \frac{(1-.88145)*10.5\,\frac{\text{g}}{\text{cm}^3}*0.6022x10^{24}\,\frac{\text{atoms}}{\text{mol}}}{15.9994\,\frac{\text{g}}{\text{mol}}} = 4.685\times10^{22}\,\frac{\text{atoms}}{\text{cm}^3}$$

$$\Sigma_t^{UO2} = N_{U235}\sigma_{U235} + N_{U238}\sigma_{U238} + N_O\sigma_O$$

$$\Sigma_t^{UO2} = 9.485\times10^{20}\left(607.5\times10^{-24}\right) + 2.248\times10^{22}\left(11.8\times10^{-24}\right) + 4.685\times10^{22}\left(3.8\times10^{-24}\right)$$

$$\Sigma_t^{UO2} = 1.02cm^{-1}$$

Example Problem 4

A research reactor has a thermal neutron flux of 10^{13} neutrons/cm^2-sec and a volume of 64,000 cm^3. If the thermal macroscopic fission cross section is 0.1 cm^{-1}, what is the power of the reactor?

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A research reactor has a thermal neutron flux of 10^{13} neutrons/cm^2-sec and a volume of 64,000 cm^3. If the thermal macroscopic fission cross section is 0.1 cm^{-1}, what is the power of the reactor?

- The fission reaction rate is

$$Fission\ rate = \Sigma_f\left(cm^{-1}\right)\phi\left(neutrons / cm^2 - \sec\right) =$$
$$= \left(0.1cm^{-1}\right)\left(10^{13} neutrons / cm^2 - \sec\right) = 10^{12}\ fissions / cm^3 - \sec$$
$$so\ we\ have\ \left(10^{12}\ fissions / cm^3 - \sec\right)\left(64,000cm^3\right) = 6.4x10^{16}\ fissions / \sec$$

- Each fission gives 200 MeV of energy and an MeV is 1.6x10-13 watt-sec

$$\left(6.4x10^{16}\ fissions / \sec\right)\left(200MeV / fission\right)\left(1.6x10^{-13} watt - \sec / MeV\right)$$
$$= 2x10^6 watts = 2\ MW$$

Example Problem 5

- What is the power produced by 1 gm of Pu-239 in a light water reactor with a thermal flux of 5 x 10^{13} neutrons/cm^2-sec? Consider a microscopic fission cross section for Pu-239 of 750 barns.
  a) 0.25 kW
  b) 0.3 kW
  c) 2.42 kW
  d) 3.04 kW

Example Problem 5

- What is the power produced by 1 gm of Pu-239 in a light water reactor with a thermal flux of 5 x 10^{13} neutrons/cm^2-sec? Consider a microscopic fission cross section for Pu-239 of 750 barns.

d) 3.04 kW

1 gm of Pu-239 has $\dfrac{\left(6.023x10^{23}\,atoms\,/\,mole\right)}{239.05\,gms\,/\,mole} = 2.52x10^{21}\,atoms\,/\,gm$

Fission cross section of Pu-239 is 750 barns so in a flux of $5x10^{13}\,neutrons/cm^2-\sec$

there are $\left(2.25x10^{21}\,atoms\right)\left(750x10^{-24}\,cm^2/atom\right)\left(5x10^{13}\,neutrons/cm^2-\sec\right) = 9.45x10^{13}\,fissions/\sec$

Each fission provides 200 MeV/fission or 3.22E-11 watt-sec/fission for

$$\left(9.45x10^{13}\,fission/\sec\right)\left(3.22x10^{-11}\,watt-\sec/fission\right) = 3{,}040\ watts$$