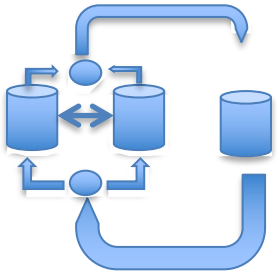

Simplified Parallel Coupled Reactors Demonstration of Regional Power Effects Including Xenon Oscillations

William Locke

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Introduction

In class, we discussed several methods used in modern reactor design calculation for the computation of space-time kinetics. These methods are complex and usually require a great deal of data for: cross-sections, material composition, thermal hydraulic behavior, and mechanical design. This paper provides a simple heuristic approach for observing phenomena which should be approachable, and which should allow the interested student to experiment further.

We set up two identical reactors and provide a pathway "pipe" for neutrons to flow between them. Each reactor is treated as a point from a kinetics perspective. The flow of neutrons between the two reactors is taken as the difference between the neutron densities (P) in the two reactors times a constant we call "g" – a pipe size. Computationally, we create a single system of two sets of six group reactor kinetics equations coupled only by the difference in the neutron densities times g. This paper does not address the actual physics related to the values for this constant.

First, we build a concept for steady state in this system. Then we consider low-power operations, and normal temperature feedback operations. Finally, we will look at xenon oscillation which may leads to the shifting of power between the two reactors in a damped or steady fashion.

This presentation explains coupling of only two reactor systems for simplicity. The approach could easily be extended to a larger system. Also, where temperature is considered, the reactors here are placed in parallel in the coolant flow.

Basic Equations

$$\frac{dP_1}{dt} = \frac{P_1(\rho_1 - \overline{\beta})}{\Lambda} + \sum_i \lambda_i \overline{C}_{1i} + g(P_2 - P_1) \qquad \frac{d\overline{C}_{1i}}{dt} = \frac{P_1 \overline{\beta}_i}{\Lambda} - \lambda_i \overline{C}_1$$

$$\frac{dP_2}{dt} = \frac{P_2(\rho_2 - \overline{\beta})}{\Lambda} + \sum_i \lambda_i \overline{C}_{2i} + g(P_1 - P_2) \qquad \frac{d\overline{C}_{2i}}{dt} = \frac{P_2 \overline{\beta}_i}{\Lambda} - \lambda_i \overline{C}_{2i}$$

Steady State

The coupling term with "g" in these equations result in a family of steady state possibilities other than only the zero-reactivity case of the decoupled equations. For example, if Reactor 1 is super critical and Reactor 2 is subcritical, there will be a power level ratio where the flow of neutrons from Reactor 1 will make up for the losses in the fission chain in Reactor 2, allowing a steady state P_1/P_2 other than one. This is similar in concept to a real reactor with an un-rodded and a rodded region. Neutrons in the rodded region flow to the un-rodded region maintaining a steady state among the two.

The steady state for our problem will be governed by the following equations:

$$\rho_{10} = \Lambda g(1 - P_{20}/P_{10}) \qquad \rho_{20} = \Lambda g(1 - P_{10}/P_{20})$$

$$\rho_{10} = \frac{-\Lambda g \rho_{20}}{\Lambda g - \rho_{20}} \qquad \rho_{20} = \frac{-\Lambda g \rho_{10}}{\Lambda g - \rho_{10}}$$

$$\rho_{10} P_{10} + \rho_{20} P_{20} = 0 \qquad P_1 \rho_1 + \Lambda g(P_2 - P_1) = 0$$

$$P_2 \rho_2 + \Lambda g(P_1 - P_2) = 0$$

The positive reactivities need to be less than or equal to Λg for a steady state to exist.

As an example, if the product Λg is 0.1 and the Reactor 1 reactivity is 500 pcm, a steady state would exist if the Reactor 2 is subcritical with a reactivity of -500.16 pcm and the ratio P_1/P_2 is 1.05. Likewise, if Reactor 1 had a reactivity of 100pcm, the Reactor 2 reactivity would be -100pcm and the power ratio would be 1.01. The reactivity expressions are nearly symmetric about the zero if the reactivities are small relative to Λg. If the Reactor 2 were a medium with K_{eff} approaching zero, $\rho_2 \implies -\infty$, Reactor 1 reactivity would need to be Λg for a steady state. At that value, Reactor 1 would have adequate neutron multiplication to sustain the loss g times P_1 getting nothing back from Reactor 2. P_2 would be zero.

It is worth noting that, if the pair of reactors starts in an initial steady state and is then perturbed in some fashion, the pair may settle at another point on the steady state curve not winding up where they started.

It is also important to recognize that this presentation is for illustration only. The actual value of a local flux will impact reactivity in a way that requires a much more complex computation, and a will provide radically different results in some circumstances.

System Equations

The dynamics of this system without temperature feedback may be shown as:

$$A=\begin{bmatrix}
\frac{\rho_1-\beta}{\Lambda}-g & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_1/\Lambda & -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_2/\Lambda & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_3/\Lambda & 0 & 0 & -\lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_4/\Lambda & 0 & 0 & 0 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_5/\Lambda & 0 & 0 & 0 & 0 & -\lambda_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_6/\Lambda & 0 & 0 & 0 & 0 & 0 & -\lambda_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
g & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\rho_2-\beta}{\Lambda}-g & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1/\Lambda & -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_2/\Lambda & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_3/\Lambda & 0 & 0 & -\lambda_3 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_4/\Lambda & 0 & 0 & 0 & -\lambda_4 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_5/\Lambda & 0 & 0 & 0 & 0 & -\lambda_5 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_6/\Lambda & 0 & 0 & 0 & 0 & 0 & -\lambda_6 & 0 & 0 & 0\\
1/Creactor & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_R & 0 & 1/\tau_R\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/Creactor & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/\tau_R & 1/\tau_R\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/\tau & 1/\tau & -2/\tau
\end{bmatrix}$$

$$X=\begin{bmatrix}
P_1\\ C_1^1\\ C_2^1\\ C_3^1\\ C_4^1\\ C_5^1\\ C_6^1\\ P_2\\ C_1^2\\ C_2^2\\ C_3^2\\ C_4^2\\ C_5^2\\ C_6^2\\ T_h^1\\ T_h^2\\ T_c
\end{bmatrix}$$

$$\frac{dX}{dt}=AX+B$$

$$\frac{dT_h^1}{dt}=\frac{(T_c-T_h^1)}{\tau_R}+P_1/C_{Reactor}\qquad C_{Reactor}=C/2$$

$$G=(I-A\Delta t)^{-1}$$

$$\frac{dT_h^2}{dt}=\frac{(T_c-T_h^2)}{\tau_R}+P_2/C_{Reactor}\qquad C_{Reactor}=C/2$$

$$\frac{\tau}{C}=1\,°F/sec$$

$$\frac{dT_c}{dt}=\frac{T_h^1+T_h^1-2T_c}{\tau}-P_{SG}/C$$

Chosen to make delta T equate to percent power.

$$X_k=G(X_{k-1}+B\Delta t)$$

Note if reactivity is changing G is not constant so it must be recomputed on a stepwise basis. If it is constant, it may be computed only once prior to the iteration.

$$Xo=\begin{bmatrix}
P_0^1\\
\beta_1P_0^1/\lambda_1\Lambda\\
\beta_2P_0^1/\lambda_2\Lambda\\
\beta_3P_0^1/\lambda_3\Lambda\\
\beta_4P_0^1/\lambda_4\Lambda\\
\beta_5P_0^1/\lambda_5\Lambda\\
\beta_6P_0^1/\lambda_6\Lambda\\
P_0^2\\
\beta_1P_0^2/\lambda_1\Lambda\\
\beta_2P_0^2/\lambda_2\Lambda\\
\beta_3P_0^2/\lambda_3\Lambda\\
\beta_4P_0^2/\lambda_4\Lambda\\
\beta_5P_0^2/\lambda_5\Lambda\\
\beta_6P_0^2/\lambda_6\Lambda\\
0\\0\\0
\end{bmatrix}$$

This solution method may be used with the expected results on a 64bit computer. Understand however that from a numerical standpoint this calculation is most demanding. The eigenvalues of the A matrix with the temperature terms included vary from zero to minus ninety-six inverse seconds. This spectral radius is large. Significant improvement in the numerical problem is gained using the prompt jump assumption in cases of small reactivity, $\Lambda\frac{dP}{dt}\approx 0$. That method adds complexity to the A matrix and

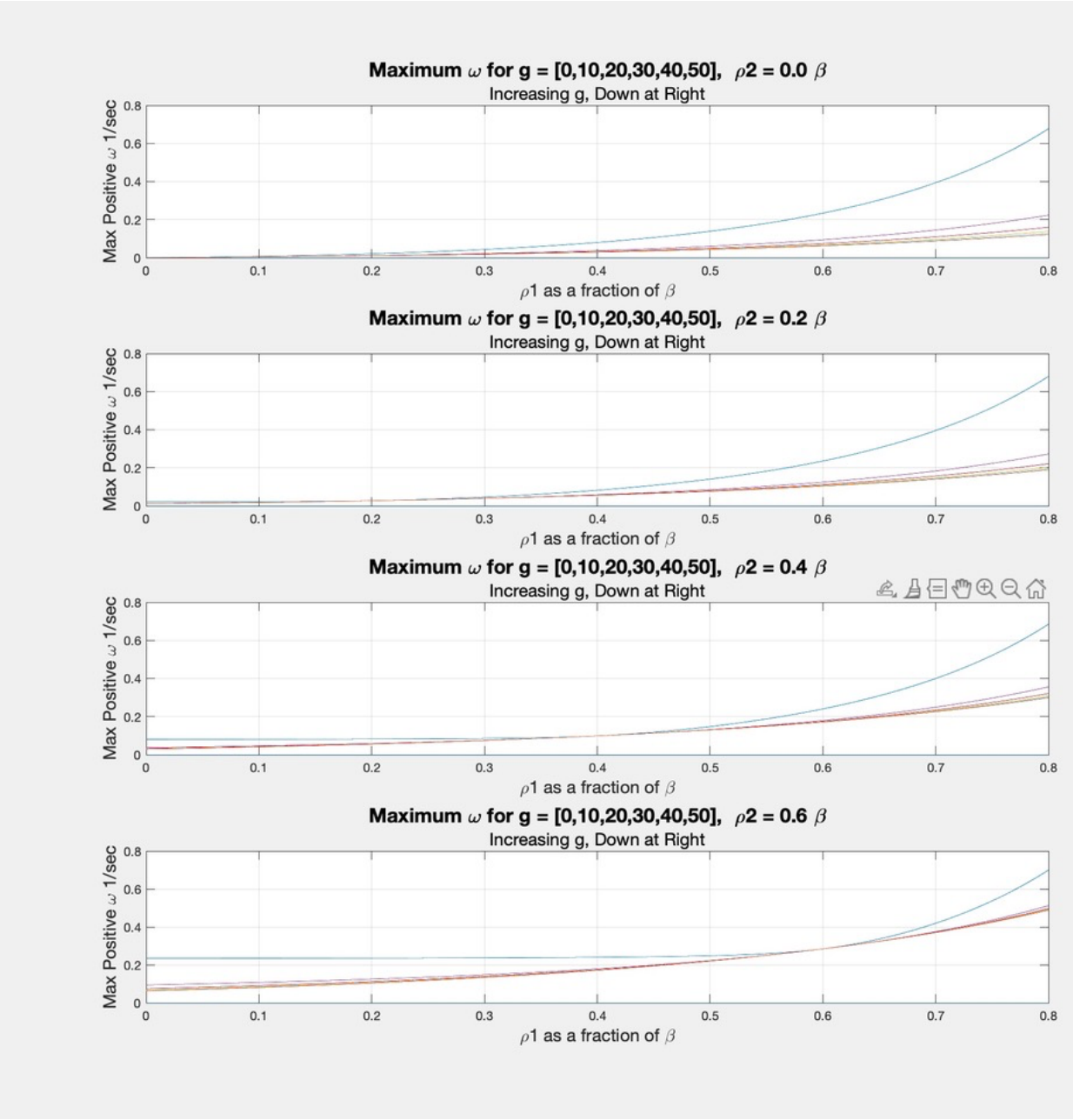
related computations. The method is discussed in the additional notes at the end of this document. Also, a more complicated $G = \left(I - \frac{A\Delta t}{2}\right)^{-1}(I + A\Delta t/2)$ would improve accuracy.

Finally, for a xenon calculation, a quasi-static method assumes that the neutron kinetics aspects of the problem are forever in steady state. This also produces excellent results because the xenon variations happen over many hours. This approach is also described in the notes at the end of this paper.

Low Power Operation

The system matrix shown above (without the temperature feedback) will have entirely real eigenvalues with a positive eigenvalue for each reactor with a positive reactivity. If only one reactor has a positive reactivity there will be only one positive eigenvalue. If both are positive with two positive eigenvalues, in time the entire system will appear to have a single stable period. The following plot shows the maximum eigenvalue for this system as a function of the reactivity in Reactor 1, with a range of values for the coupling, g, and for several different values of reactivity in Reactor 2. The maximum eigenvalue will become the stable period with positive reactivities.

Maximum Eigenvalue Plot



This plot shows the impact of the sharing of neutrons as a function of g. If Reactor 2 has zero reactivity, the largest eigenvalue will drop for high reactivities in Reactor 1 as g increases – neutrons are flowing from Reactor 1 to Reactor 2. In the case where Reactor 2 has a high positive reactivity, the system will have a strong positive eigenvalue even when the Reactor 1 has zero reactivity.

The Prompt Jump

A single reactor, non-coupled, which is initially critical and provided a reactivity step, has a prompt jump given by $P_o\beta/(\beta-\rho)$. This is known as the prompt jump equation. With the coupled reactor pair, a reactivity insertion in one of the reactors will also cause a prompt jump but calculation of its size is more complex because some of the neutrons are escaping to the other reactor as the prompt neutron equilibrium is being established. We consider the special case with an initial condition that both reactors have the same initial power level P_o and both have a zero reactivity. Insert a reactivity, ρ, into Reactor 1. In that case the jump will be given for both reactors as follows:

$$P_1 = P_0 \frac{\overline{\beta}(\overline{\beta} + 2g\Lambda)}{\overline{\beta}^2 + 2g\Lambda - \rho(\overline{\beta} + g\Lambda)}$$

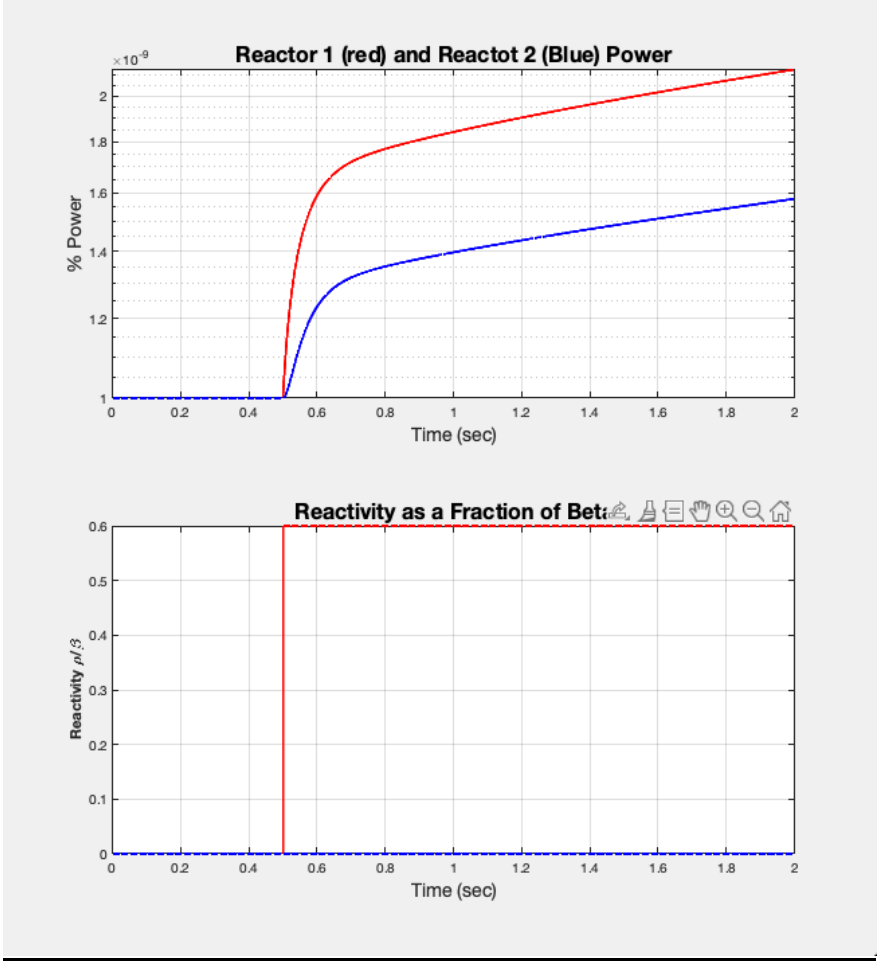
Step in Reactor 1 only

$$P_2 = P_0 \frac{\overline{\beta}(\overline{\beta} + 2g\Lambda - \rho)}{\overline{\beta}^2 + 2g\Lambda - \rho(\overline{\beta} + g\Lambda)}$$

$P_{10} = P_{20} = P_0$

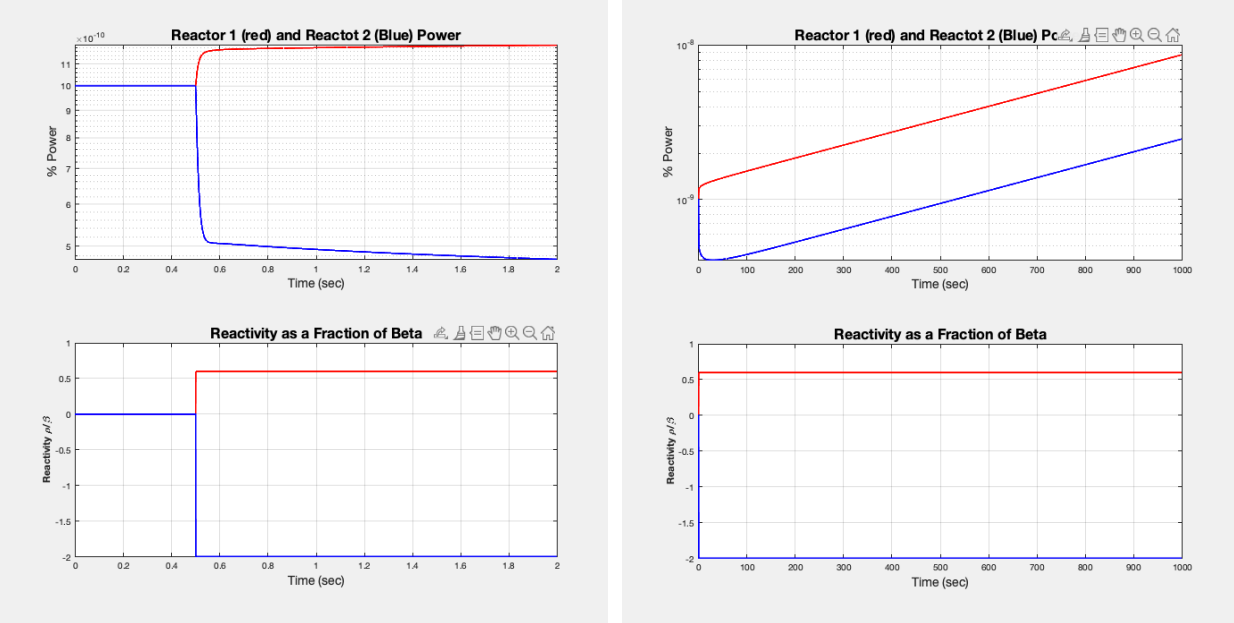
The plot below demonstrates a step into reactor one of 0.6β, the power in both reactors responds as predicted (g = 20, Λ = 2.6 10^{-4} sec):

Power and Reactivity vs. Time for a Reactivity Step



The next plot demonstrates the behavior in the system if both reactors are given a step: Reactor 1 is given a step of +0.6β and Reactor 2 is given a step of -2.0β. The calculation of the prompt jump in this case is more complicated than given by the equations on the last page. (g = 20, Λ = 2.6 10^{-4} sec)

Power and Reactivity vs. Time for a Double Reactivity Step



These plots are particularly interesting because they demonstrate that while Reactor 2 initially appears to be shutdown, there is adequate excess capability in Reactor 1 to drive the power up over time in both reactors. The calculated final startup rate for each of the two reactors is 0.05 DPM (20 minutes per decade). The Reactor 1 reactivity value is 0.6β, a significant positive reactivity normally leading to a large startup rate. The other reactor, with a reactivity of -2.0β, represents a significant drag on the system. The combination yields the low startup rate.

Power Operations (Temperature Feedback)

With temperature feedback incorporated into this system, improved coordination is obtained. The transient below is started with the initial powers forced apart (70% and 30%). The temperature feedback draws them together. Once that happens, (at 100 sec), reactor 1 has a negative reactivity inserted equal to -0.5β. We see Reactor 2 maneuver to pick up the load and carry on alone. The Steam Demand remains 100%. The temperature in reactor 1 is dropped by 18°F amounting to a reactivity increase of about 0.3 β. The flow of neutrons out of Reactor 2 to Reactor 1 accounts for an apparent reactivity drop in reactor 2 compensated for by a temperature drop of 8.9°F. In the end $P_1\rho_1 + P_2\rho_2 \approx 0$. The fuel reactivity effects are not included here.

Note that if two reactors are in steady state and then get joined by a coupling factor g, there will be a prompt jump or drop among the pair as shown here. After the jump the powers will be as follows:

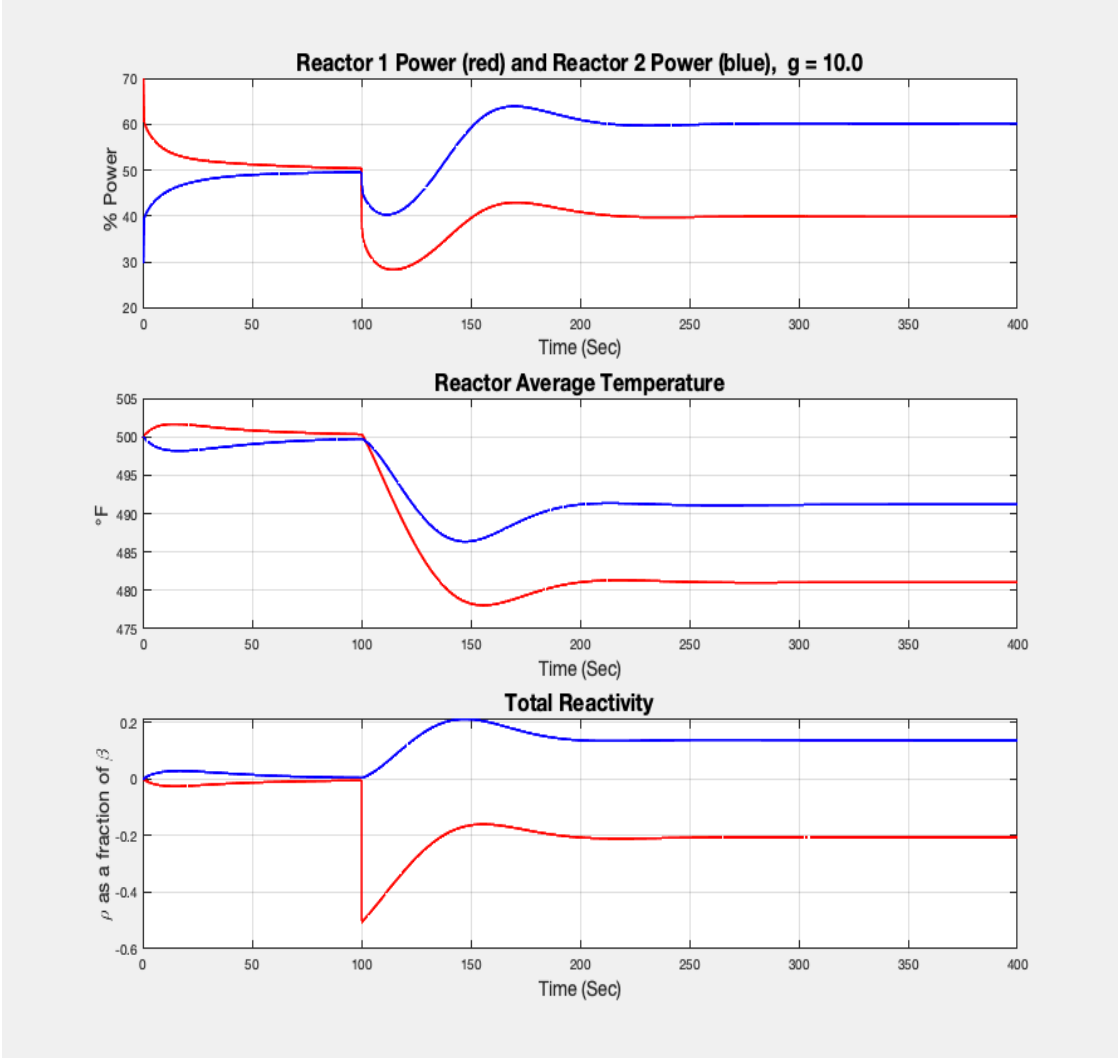
$$P_1 = \frac{(P_{01} + P_{02})g\Lambda + P_{01}\beta}{\beta + 2g\Lambda} \qquad P_2 = P_{SG} - P_1$$

P_1 = 60.16 % $\qquad$ P_2 = 39.84 %

When the reactors are first joined, they will exhibit an initial startup rate in each reactor as follows. This is fictitious because it relates to a sudden connection between two reactors which is non-physical.

$$SUR_1 = 26.06 dpm - sec\frac{g(P_{02} - P_{01})}{P_{01}} \qquad SUR_1 = -149\ dpm$$

$$SUR_2 = 26.06 dpm - sec\frac{g(P_{01} - P_{02})}{P_{02}} \qquad SUR_2 = 347\ dpm$$


Reactor 1 Power (red) and Reactor 2 Power (blue), g = 10.0
% Power
Time (Sec)
Reactor Average Temperature
°F
Time (Sec)
Total Reactivity
ρ as a fraction of β
Time (Sec)

Incorporation of Xenon

We will now modify our problem to track Xenon separately in each reactor. We previously developed the following constants and equations:

I-135 fission yield	γ_I	5.7%
Xe-135 fission yield	γ_{Xe}	0.3%
I-135 decay constant (6.7 hour $t_{1/2}$)	λ_I	2.87e-05 sec^{-1}
Xe-135 decay constant (9.2 hour $t_{1/2}$)	λ_{Xe}	2.09e-05 sec^{-1}
Full Power Burnout Factor $\sigma_a^{Xe}\phi_{th}^{100\%}$, the value of R^{Max} varies with core life as the flux changes for a particular power.	R^{Max}	7.34e-05 sec^{-1}
Power Constant based on a Full Power Equilibrium Xe Reactivity of -2900 pcm (Based on SNUPS Reactors).	K	-4.56 pcm-sec^{-1}

$$B = pK\begin{bmatrix}\gamma_I \\ \gamma_{Xe}\end{bmatrix} \qquad A = \begin{bmatrix}-\lambda_I & 0 \\ \lambda_I & -\lambda_{Xe} - R^{Max}p\end{bmatrix} \qquad N_{Xe}^{Eq} = \frac{(\gamma_{Xe}+\gamma_I)pK}{\lambda_{Xe}+pR^{Max}} \qquad p = P/100$$

$$\frac{dX}{dt} = AX + B \qquad X = \begin{bmatrix}N_I \\ N_{Xe}\end{bmatrix}$$

$$\frac{X_k - X_{k-1}}{\tau_{step}^{[\sec]}} = AX_k + B \qquad X_k = G(X_{k-1} + B\tau_{step}^{[\sec]})$$

$$G = (I - A\tau_{step}^{[\sec]})^{-1}$$

$$G = \begin{bmatrix} \frac{1}{1+\lambda_I\tau_{step}^{[\sec]}} & 0 \\ \frac{\lambda_I\tau_{step}^{[\sec]}}{(1+\tau_{step}^{[\sec]}(\lambda_{Xe}+R^{Max}p))(1+\lambda_I\tau_{step}^{[\sec]})} & \frac{1}{1+\tau_{step}^{[\sec]}(\lambda_{Xe}+R^{Max}p)} \end{bmatrix}$$

The simple form of G here is taken because the half-lives associated with Xe and I are much longer than the time step being used for the kinetics.

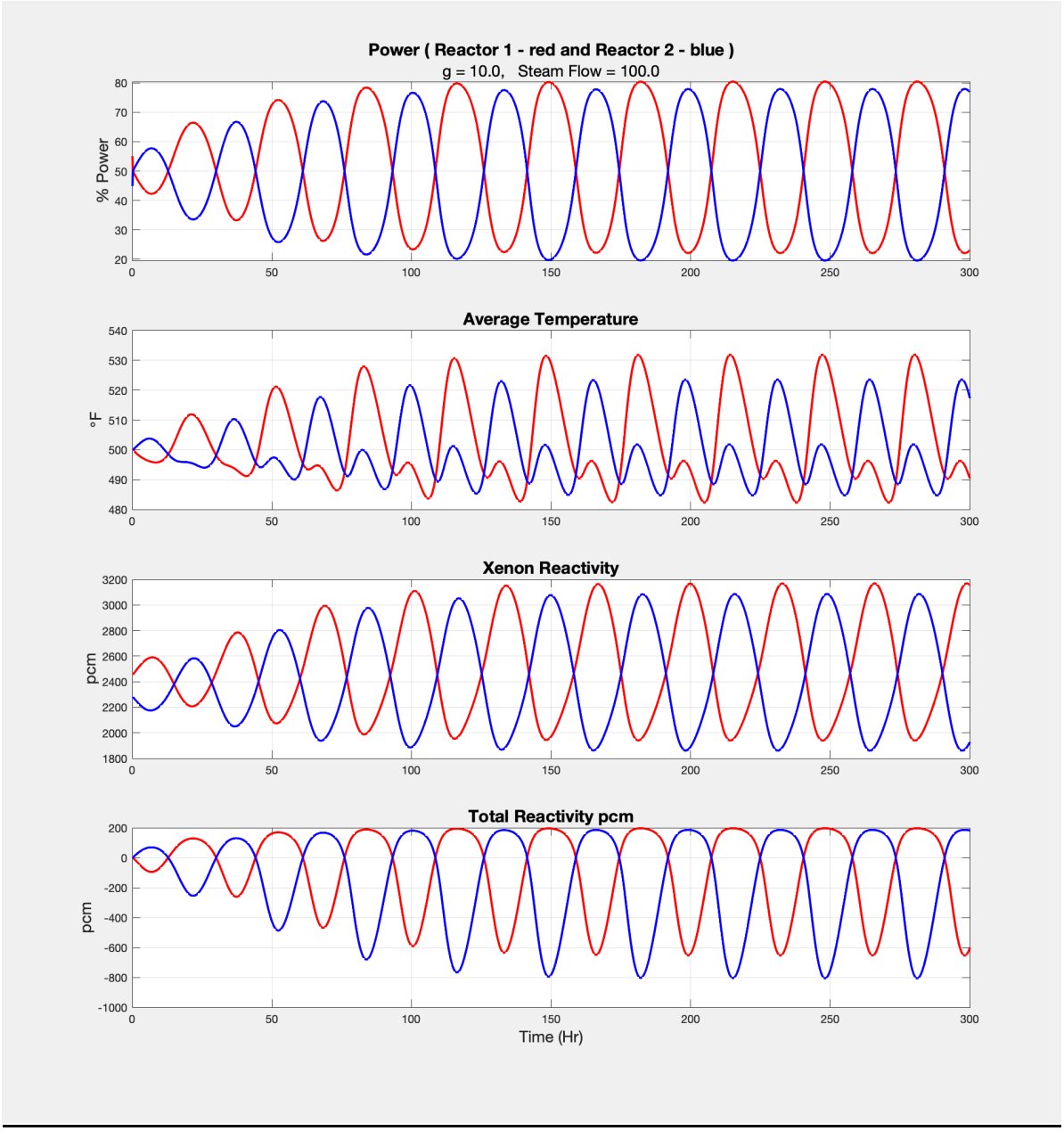
The calculation is performed for several steps with the reactor kinetics equations and the resulting transient temperature. Then the xenon and iodine concentrations are found. The new xenon reactivity information is used to update the kinetics matrix for the next kinetics steps. Both Xenon and kinetics (prompt jump approximation) propagation matrixes are being recalculated for each inner or outer step.

Oscillation is setup by demanding 100% steam flow and misadjusting the initial conditions. Reactor 1 is started at a steady state 55% power and equilibrium xenon. Reactor 2 is likewise started at 45% power and equilibrium xenon. The reactors are then connected with g at various values. The plots on the next three pages show first a case where the value of g = 10. This results in a strong sustained oscillation. The subsequent plots show the results with larger values of g.

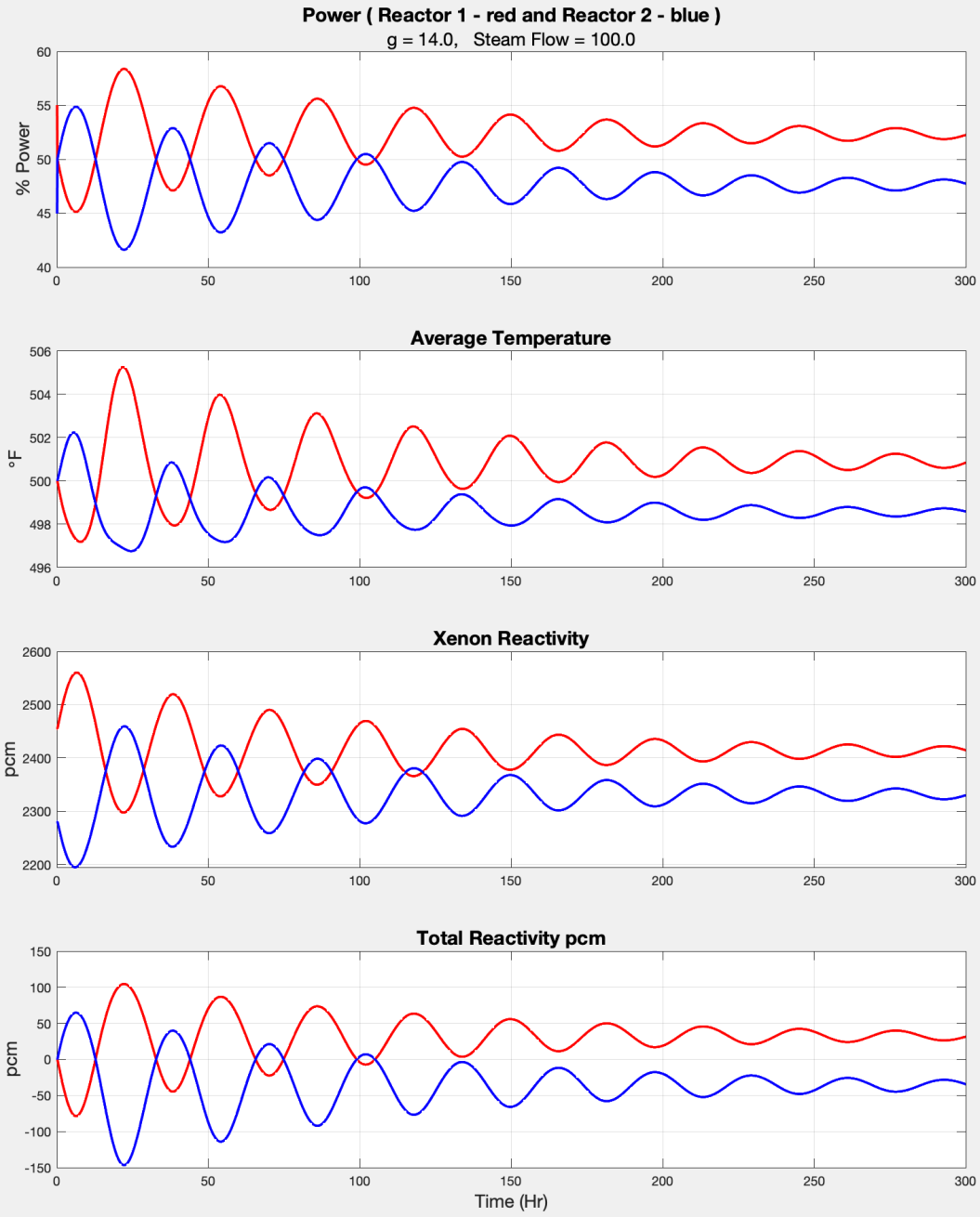
Xenon Oscillations

The purpose of this coupled reactor exercise has been to explore some space-time concepts without a great deal of detailed mathematics and computer time. The ideas presented here are like what are found in practice, but the detail has been passed over for the purpose of instruction. These examples assume high enrichment with $\alpha_F = 0$. Another set below show the results including α_F not equal to zero.

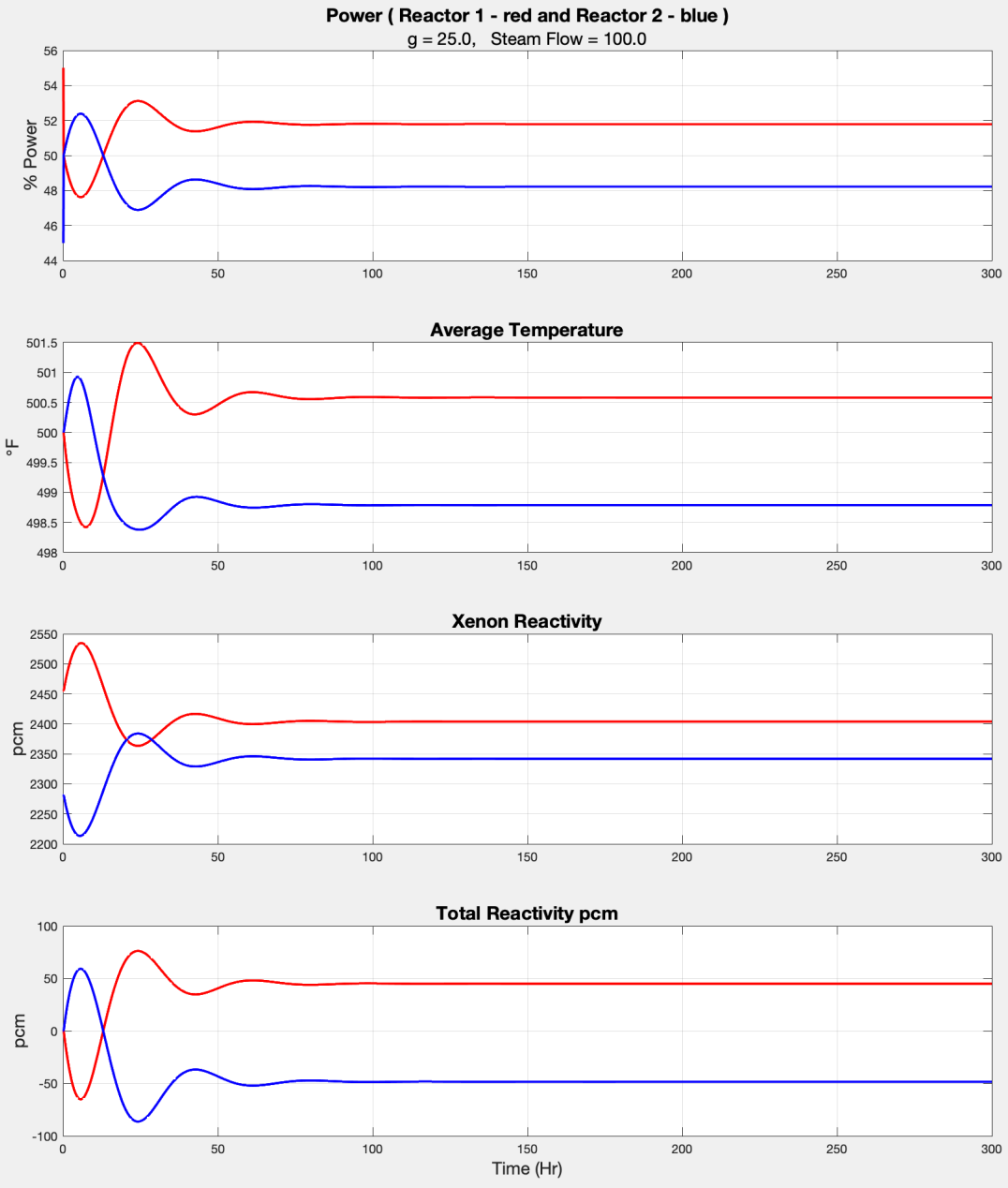
Strong Xenon Oscillations between Reactor 1 and Reactor 2



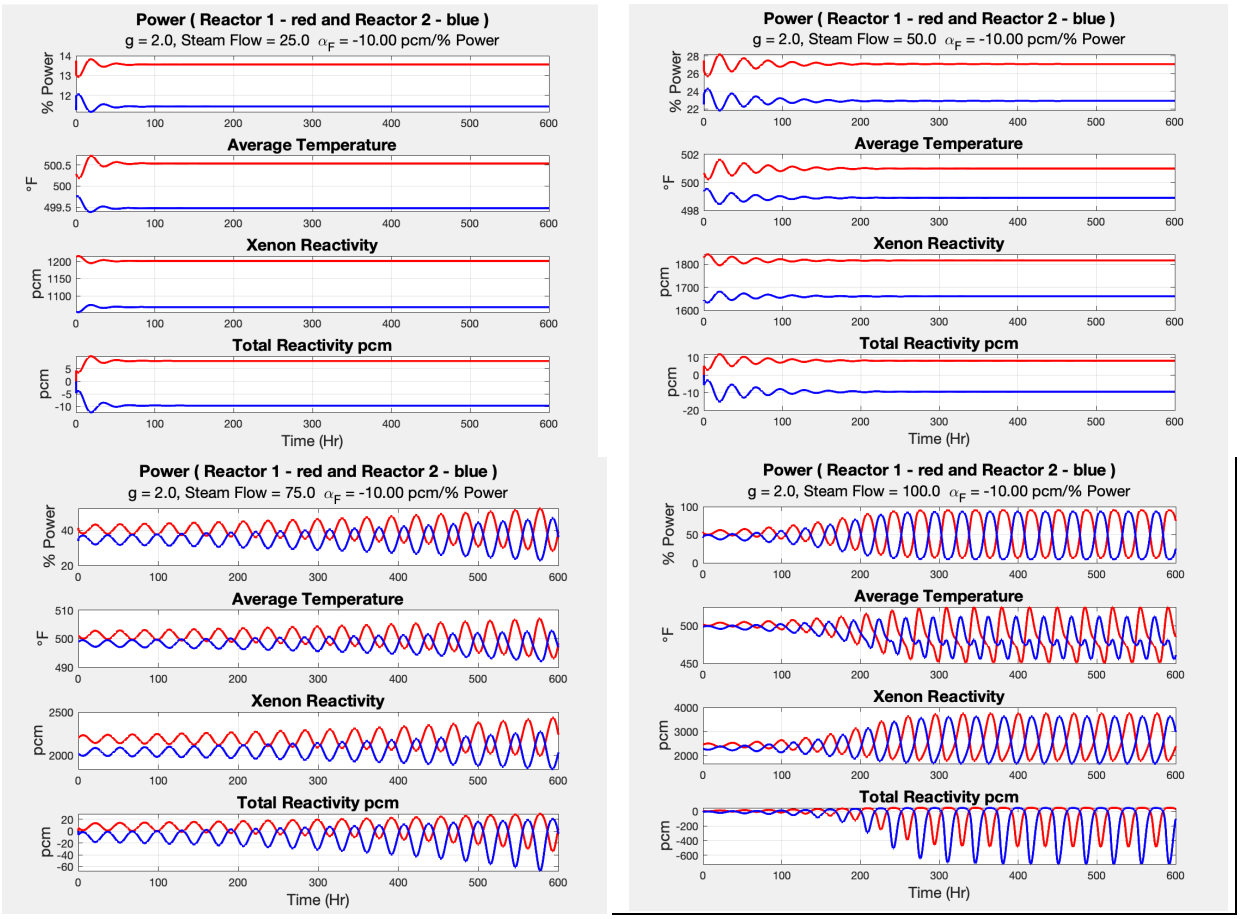
Damped Xenon Oscillations between Reactor 1 and Reactor 2



Strongly Damped Xenon Oscillations between Reactor 1 and Reactor 2



The amount of oscillation depends on the percent steam flow as well as the coupling constant g. This is true whether the fuel temperature feedback (power reactivity) is included in the computation or not. The following four cases do include the power reactivity and they appear in increasing order of steam flow. The power reactivity does provide significant damping, so a smaller g is required in that case to see the oscillations.



Methods used for computing these curves with the inclusion of the power reactivity are discussed in the notes provided below.

Additional Notes:

1. <u>The One-delayed neutron group coupled reactor period equation:</u>

$$\mathrm{T} = \overline{\beta}\left[\frac{2g\Lambda - \rho_1 - 2g\Lambda\rho_1/\overline{\beta} - \rho_2 - 2g\Lambda\rho_2/\overline{\beta} + \rho_1\rho_2/\overline{\beta} \pm \sqrt{4g^2\Lambda^2 + \rho_1^{\,2} - 2\rho_1\rho_2 + \rho_2^{\,2}}}{2\lambda_{eff}(g\Lambda\rho_1 + g\Lambda\rho_2 - \rho_1\rho_2)}\right]$$

   Derived by assuming a stable period for the system, $\frac{P_1}{\dot{P}_1} = \frac{P_2}{\dot{P}_2} = T$ and $P_2 = K P_1$. This will only make sense once the stable period is established. The square root term results from a quadratic solution. Care must be taken to select the correct sign. In most cases selection is trivial but in some cases it may not be trivial.

   This equation suffers the usual ills of the one delayed group model. λ_{eff} is a function of the reactor period and hence the estimate needs its own answer to be accurate.

2. <u>Generalized Prompt Jump Expression:</u>

$$P_1 = \frac{\overline{\beta}(\overline{\beta} + g\Lambda - \rho_2)P_{01} + \overline{\beta}g\Lambda P_{02}}{(\beta - \rho_1)(\beta - \rho_2) + g\Lambda(\beta - \rho_1) + g\Lambda(\beta - \rho_2)}$$

$$P_2 = \frac{\overline{\beta}(\overline{\beta} + g\Lambda - \rho_1)P_{02} + \overline{\beta}g\Lambda P_{01}}{(\beta - \rho_1)(\beta - \rho_2) + g\Lambda(\beta - \rho_1) + g\Lambda(\beta - \rho_2)}$$

   In these expressions P_{01} and P_{02} are initial steady state values based on the initial reactivities in the two reactors. The expression assumes that the reactivities ρ_1 and ρ_2 are inserted at the same time in Reactor 1 and Reactor 2.

   Given a steady state P_{01} and an initial reactivity ρ_{10}, the initial power and reactivity in the other reactor are given by:

$$P_{20} = P_{10}(1 - \rho_{10}/g\Lambda)$$
$$\rho_{20} = \frac{-\Lambda g\rho_{10}}{\Lambda g - \rho_{10}} \qquad \rho_{10} < g\Lambda$$

3. <u>Calculation using Prompt Jump Assumption without fuel feedback from α_F.</u>

This method assumes that the governing equations may be taken as the following:

$$0 = P_1(\beta - \rho_1) + \Lambda K_1 + \Lambda g(P_2 - P_1) \qquad K_1 = \sum_{i=1}^{6} \lambda_i C_{1i} \qquad \frac{dC_{1i}}{dt} = \frac{\beta_i P_1}{\Lambda} - \lambda_i C_{1i}$$

$$0 = P_2(\beta - \rho_2) + \Lambda K_2 + \Lambda g(P_1 - P_2) \qquad K_2 = \sum_{i=1}^{6} \lambda_i C_{2i} \qquad \frac{dC_{2i}}{dt} = \frac{\beta_i P_2}{\Lambda} - \lambda_i C_{2i}$$

The approach we will take is have the differential equation solve for the precursor concentrations in both reactors as well as the$T_h^1, T_h^2, and\ T_c$. We will then compute the power from these precursor concentrations as we did in the usual prompt jump approximation for a single reactor: $P = \frac{\Lambda \sum_i \lambda_i C_i}{\beta - \rho}$

Solving the above equations for the power in terms of the precursor sums yields the following assuming α_F is taken as zero.

$$P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \Gamma \begin{bmatrix} \Lambda K_1 \\ \Lambda K_2 \end{bmatrix} \qquad \Gamma = \frac{\begin{bmatrix} \beta - \rho_2 + \Lambda g & \Lambda g \\ \Lambda g & \beta - \rho_1 + \Lambda g \end{bmatrix}}{(\beta - \rho_2 + \Lambda g)(\beta - \rho_1 + \Lambda g) - (\Lambda g)^2}$$

$$\frac{dT_h^1}{dt} = \frac{(T_c - T_h^1)}{\tau_R} + P_1/C_{Reactor} \qquad \frac{dT_h^2}{dt} = \frac{(T_c - T_h^2)}{\tau_R} + P_2/C_{Reactor}$$

$$\frac{dT_c}{dt} = \frac{T_h^1 + T_h^1 - 2T_c}{\tau} - P_{SG}/C$$

Using this, the system matrix becomes:

$$Am = \begin{vmatrix} \beta_1\lambda_1 & \beta_1\lambda_2 & \beta_1\lambda_3 & \beta_1\lambda_4 & \beta_1\lambda_5 & \beta_1\lambda_6 \\ \beta_2\lambda_1 & \beta_2\lambda_2 & \beta_2\lambda_3 & \beta_2\lambda_4 & \beta_2\lambda_5 & \beta_2\lambda_6 \\ \beta_3\lambda_1 & \beta_3\lambda_2 & \beta_3\lambda_3 & \beta_3\lambda_4 & \beta_3\lambda_5 & \beta_3\lambda_6 \\ \beta_4\lambda_1 & \beta_4\lambda_2 & \beta_4\lambda_3 & \beta_4\lambda_4 & \beta_4\lambda_5 & \beta_4\lambda_6 \\ \beta_5\lambda_1 & \beta_5\lambda_2 & \beta_5\lambda_3 & \beta_5\lambda_4 & \beta_5\lambda_5 & \beta_5\lambda_6 \\ \beta_6\lambda_1 & \beta_6\lambda_2 & \beta_6\lambda_3 & \beta_6\lambda_4 & \beta_6\lambda_5 & \beta_6\lambda_6 \end{vmatrix} \qquad L = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6 \end{bmatrix}$$

Note: This is the tensor outer product $\vec{\beta} \otimes \vec{\lambda}$.

$$A^{15x15} = \begin{bmatrix} \Gamma_{11}Am - L & \Gamma_{12}Am & 0 & 0 & 0 \\ \Gamma_{21}Am & \Gamma_{22}Am - L & 0 & 0 & 0 \\ \Lambda\Gamma_{11}Lm/C_R & \Lambda\Gamma_{21}Lm/C_R & -1/\tau_R & 0 & 1/\tau_R \\ \Lambda\Gamma_{12}Lm/C_R & \Lambda\Gamma_{22}Lm/C_R & 0 & -1/\tau_R & 1/\tau_R \\ 0 & 0 & 1/\tau & 1/\tau & -2/\tau \end{bmatrix} \qquad Lm = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6]$$

$$\frac{dX}{dt} = A(X)X + B$$

A is a function of X and its history because A is a function of the reactivities. Hence this is a nonlinear system.

$$B = \frac{-P_{SG}}{C_{Heat\ Capacity}} \begin{bmatrix} 0^{14x1} \\ 1 \end{bmatrix}$$

$$X^{15x1} = [C_{1i=1:6} \quad C_{2i=1:6} \quad T_h^1 \quad T_h^2 \quad T_c]^T$$

$$Xo^{15x1} = \left[\frac{P_{10}\beta_{i=1:6}}{\Lambda\lambda_{i=1:6}} \quad \frac{P_{20}\beta_{i=1:6}}{\Lambda\lambda_{i=1:6}} \; 0 \; 0 \; 0\right]^T$$

The eigenvalues of this system range between 0 and -3 sec^{-1} which is a significant improvement over the full kinetics version of the solution.

4. One Delayed Group Prompt Jump Approximation with Power Reactivity

The transient involved with xenon oscillation happens over many hours. This means that the startup rates associated with the oscillation is very small. As such λ_{eff} is nearly constant at a value of 0.077 1/sec. In this case a single delayed group may be used to produce results which are nearly identical to the results found in the more complex calculations. The prompt jump version of the solution reduces to the following in that case:

$$\Gamma = \frac{\begin{bmatrix} \beta - \rho_2 + \Lambda g & \Lambda g \\ \Lambda g & \beta - \rho_1 + \Lambda g \end{bmatrix}}{(\beta - \rho_2 + \Lambda g)(\beta - \rho_1 + \Lambda g) - (\Lambda g)^2} \qquad P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \Gamma \begin{bmatrix} \Lambda\lambda_{eff} C_1 \\ \Lambda\lambda_{eff} C_2 \end{bmatrix}$$

$$A^{3x1} = \begin{bmatrix} \Gamma_{11}\beta\lambda_{eff} - \lambda_{eff} & \Gamma_{12}\beta\lambda_{eff} & 0 & 0 & 0 \\ \Gamma_{21}\beta\lambda_{eff} & \Gamma_{22}\beta\lambda_{eff} - \lambda_{eff} & 0 & 0 & 0 \\ \Lambda\Gamma_{11}\lambda_{eff}/C_R & \Gamma_{21}\lambda_{eff}/C_R & -1/\tau_R & 0 & 1/\tau_R \\ \Gamma_{12}\lambda_{eff}/C_R & \Lambda\Gamma_{22}\lambda_{eff}/C_R & 0 & -1/\tau_R & 1/\tau_R \\ 0 & 0 & 1/\tau & 1/\square\square & -2/\tau \end{bmatrix}$$

$$\frac{d\delta T_{Ave}}{dt} = \frac{P_1 + P_2}{C_{Heat\ Capacity}} - \frac{P_{SG}}{C_{Heat\ Capacity}}$$

$$\frac{dX}{dt} = A(X)X + B$$

A is a function of X and its history because A is a function of the reactivities. Hence this is a nonlinear system.

$$B = \frac{-P_{SG}}{C_{Heat\ Capacity}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$X^{2x1} = [C_1 \quad C_2 \quad T_h^1 \quad T_h^2 \quad T_c]^T$$

$$Xo^{12x1} = \begin{bmatrix} \frac{P_{10}\beta}{\Lambda\lambda_{eff}} & \frac{P_{20}\beta}{\Lambda\lambda_{eff}} & 0 & 0 & 0 \end{bmatrix}^T$$

5. Power Solution with Prompt Jump Assumption and fuel feedback from α_F.

This case is more difficult than the no fuel feedback case.

$0 = P_1(\rho_1 + \alpha_F P_1 - \beta) + \Lambda K_1 + \Lambda g(P_2 - P_1)$

ρ_1 Includes effects from temperature and xenon.

$$K_1 = \sum_{i=1}^{6} \lambda_i C_{1i} \qquad \frac{dC_{1i}}{dt} = \frac{\beta_i P_1}{\Lambda} - \lambda_i C_{1i}$$

$0 = P_2(\rho_2 + \alpha_F P_2 - \beta) + \Lambda K_2 + \Lambda g(P_1 - P_2)$

ρ_2 Includes effects from temperature and xenon.

$$K_2 = \sum_{i=1}^{6} \lambda_i C_{2i} \qquad \frac{dC_{2i}}{dt} = \frac{\beta_i P_2}{\Lambda} - \lambda_i C_{2i}$$

One may solve these equations to develop a fourth order polynomial in P_1 which may be solved, or one may use an equation solver to solve for both values directly with an initial estimate being the most recent value of P1 and P2.

The polynomial in Power is as follows:

$$a_0 P_1^4 + a_1 P_1^3 + a_2 P_1^2 + a_3 P_1 + a_4$$

$$a_0 = {\alpha_F}^3 / \Lambda^2 g^2$$

$$a_1 = 2{\alpha_F}^2(-\Lambda g + \rho_1 - \beta)/\Lambda^2 g^2$$

$$a_2 = \alpha_F(2\Lambda^2 g^2 + 3\Lambda\beta g - 2\Lambda g\rho_1 - \Lambda g\rho_2 + 2K_1\alpha_F\Lambda + (\beta - \rho_1)^2)/\Lambda^2 g^2$$

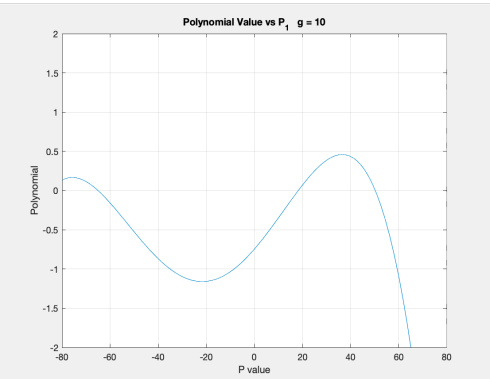
$$a_3 = \rho_1 - \beta - \left(\frac{(\beta - \rho_1 + \Lambda g)(K_1\alpha_F + \beta g - g\rho_2)}{\Lambda g^2}\right) - \frac{K_1\alpha_F(\beta - \rho_1 + \Lambda g)}{\Lambda g^2}$$

$$a_4 = \Lambda K_1 + \Lambda K_2 + \frac{K_1(\beta - \rho_2 + K_1\alpha_F/g)}{g}$$

$$P_2 = -\frac{1}{\Lambda g}(P_1(\rho_1 + \alpha_F P_1 - \beta) + \Lambda K_1 - \Lambda g(P_1)$$

Once P_1 is obtained, find P_2 with this equation.

This polynomial may have four real roots. In the cases observed, the root of choice is the largest. Both methods appear to work. The most reliable would be the direct solve method. The root finding method is faster.



6. Steady State Criteria

a. The steady state for the coupled reactor and steam plant system will require the following conditions to be met. That said if the system has perpetual xenon oscillations these conditions will never be met.:

$$P_1 + P_2 = P_{SteamSG}$$

$$T_h^1 = T_c + P_1 \tau_R / C_{reactor}$$

$$T_h^2 = T_c + P_2 \tau_R / C_{reactor}$$

$$\rho_{Xe}^1 = \frac{(\gamma_{Xe} + \gamma_I) p_1 K}{\lambda_{Xe} + p_1 R^{Max}} \qquad p_1 = P_1/100$$

$$\rho_{Xe}^2 = \frac{(\gamma_{Xe} + \gamma_I) p_2 K}{\lambda_{Xe} + p_1 R^{Max}} \qquad p_2 = P_2/100$$

$$\rho_1 = -\rho_{Xe}^{1\,initial} + \rho_{Xe}^1 + \alpha_w \left[\frac{T_h^1 + T_c}{2} - T_{Reference} \right] + \alpha_F (P_1 - P_{10})$$

$$\rho_2 = -\rho_{Xe}^{2\,initial} + \rho_{Xe}^2 + \alpha_w \left[\frac{T_h^2 + T_c}{2} - T_{Reference} \right] + \alpha_F (P_2 - P_{20})$$

$$P_1 \rho_1 + g\Lambda(P_2 - P_1) = 0$$

$$P_2 \rho_2 + g\Lambda(P_1 - P_2) = 0$$

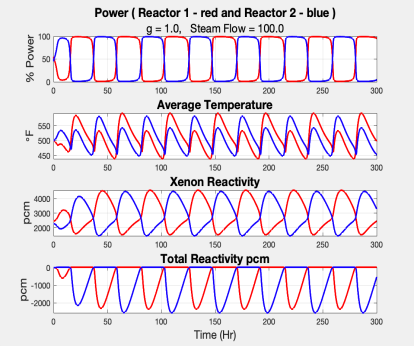
$\rho_{Xe}^{1,2\,initial}$ are the initial equilibrium xenon levels for reactor 1 and 2

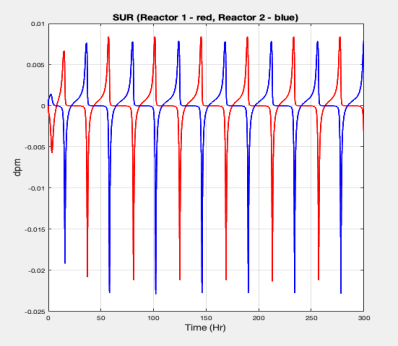
P_{10} and P_{20} are the initial powers in reactor one and two.

b. These may be directly solved by eliminating P_2 and then solving for P_1 and T_c by minimizing the right-hand side of the last two equations. With MATLAB this may be done using fsolve().

7. A Quasi-Static method for Xenon Oscillation Observation

a. We previously noted that with the rate of the xenon oscillation is low and the single delayed group kinetics model performed as well as the six-group solution method. A direct computation of the startup rates associated with these transients shows that the reactor is very near steady state. For example, with an extremely decoupled pair of reactors (g = 1) a computation of the usual data as well as the startup rate yields the following:





The startup rate varies between less than 0.01 dpm to -0.025 dpm.

This result points to an entirely different approach. Perform the xenon transient calculation assuming that the reactor kinetics are constantly in equilibrium. In other words, the steady state kinetics equations at the start of this paper hold always during the transient. Using the steady state reactor powers at each time step to compute a new xenon value, and then use that new value to compute new steady state powers.

We start with equilibrium Iodine and Xenon values in reactors with no connection to each other, g = 0. For example, with one reactor at 55% and the other at 45% power and a total steam flow of 100%.

Now connect the two reactors with g not equal zero and calculate the resultant Xenon reactivities over time following the connection.

We will use an ODE solver to solve the following equations:

[P_1 and P_2] = Compute Power

$$\frac{dN_I^1}{dt} = \gamma_I K P_1/100 - \lambda_I N_I^1$$

$$\frac{dN_{Xe}^1}{dt} = \gamma_{Xe} K P_1/100 + \lambda_I N_I^1 - \lambda_{Xe} N_{Xe}^1 - R^{Max} P_1/100$$

$$\frac{dN_I^2}{dt} = \gamma_I K P_2/100 - \lambda_I N_I^2$$

$$\frac{dN_{Xe}^2}{dt} = \gamma_{Xe} K P_2/100 + \lambda_I N_I^2 - \lambda_{Xe} N_{Xe}^2 - R^{Max} P_2/100$$

Where "Compute Power" uses the equations below. These equations are best solved numerically. Equations numbered 1 and 2 are solved together for zero roots after making the substitutions for the associated values using the rest of the equations. This solves for P_1 (and T_C). P_2 is computed from P_1 using equation 3.

1 $P_1\rho_1 + \Lambda g(P_2 - P_1) = 0$

2 $P_2\rho_2 + \Lambda g(P_1 - P_2) = 0$

3 $P_1 + P_2 = P_{Steam}$

4 $\rho_1 = Inject_1 + \rho_{Xe}^1 + \alpha_w(T_{Ave}^1 - T_{Reference}) + \alpha_F(P_1 - P_{10})$

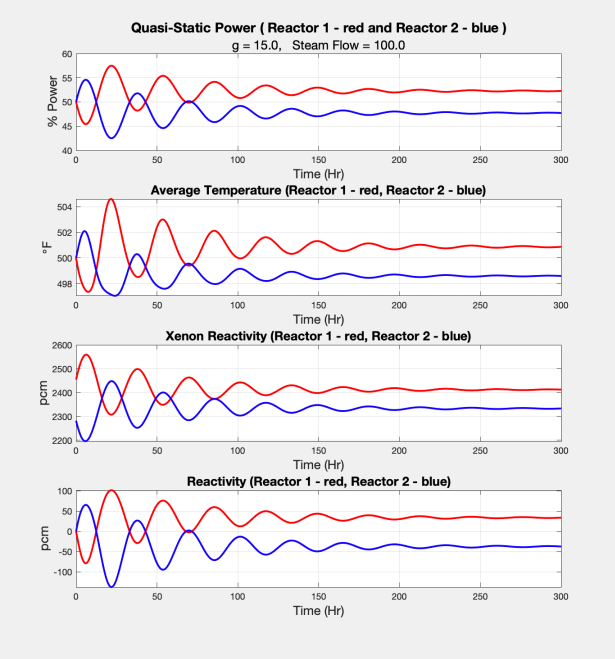
5 $\rho_2 = Inject_1 + \rho_{Xe}^2 + \alpha_w(T_{Ave}^2 - T_{Reference}) + \alpha_F(P_2 - P_{20})$

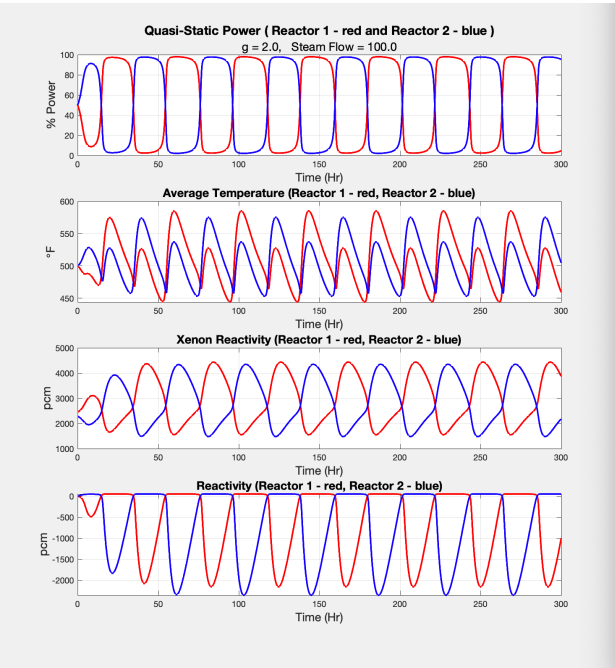
6 $T_{Ave}^1 = T_C + P_1\tau_R/(2C_R)$

7 $T_{Ave}^2 = T_C + P_2\tau_R/(2C_{R)}$

b. The quantities $Inject_1$ and $Inject_2$ are the reactivities in the reactor initially to counteract the equilibrium xenon values. P10 and P20 are the initial equilibrium power values.
c. Once the xenon differential equations are solved for a specified time integral, the values of P, Tave, and Reactivities, may be obtained by using the equations 1-7 above for each reactor iterating overall the entire time interval.

Quasi-Static solution





Six Group Prompt Jump Assumption Method